Essays on Asymmetries in Contest

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To Charlotte and Micha

Abstract

This thesis is concerned with the effects of asymmetries in ability and social preferences in contests and conflict networks. Standard models find that asymmetries monotonically decrease total and individual efforts. I demonstrate that this result does not necessarily hold when players are embedded in complex networks, have preferences regarding the fairness of the contest or the outcomes of others, and when real subjects play these games in the lab.

Chapter 1 formulates a network of bilateral contests in which locally unique equilibria always exist, and global uniqueness is possible. I find that an increase of one player's ability can increase her effort and the effort of the entire network. If one player targets a specific opponent, other players follow.

Chapter 2 imposes a budget constraint on this model. Most findings are robust to this modelling choice. This allows an investigation of topics like the effects of heterogeneity on team performance and the effect of asymmetries in the number of conflicts a player and her rivals are involved in.

Chapter 3 documents that there exists no agreed way for implementing social preferences in contests. I derive four possible versions and critically assess their properties. When costs are considered, the magnitude of predicted overspreading and overbidding is reduced. Mild asymmetry can result in higher effort from the high ability player.

In chapter 4, I present a pilot experiment in which social identities, with and without a hierarchy, are induced. We find that identities with such a hierarchy can trigger more aggressive play. To structure these findings, I suggest a foundational model of social preferences that relates them to social identity, where 'close' players are treated with altruism and 'distant' players are treated with spite.

Contents

List of Figures

List of Tables

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Introduction

In conflicts and competitions, it matters who is facing whom. This can affect how dear winning and how existential losing is. In conflict, the strength of our opponents determines how much damage they can bring about. In a competitive environment, their ability matters, as they might have higher chances to get a job, a promotion or the winning prize in a sports competition. If rivals are of lower social status, losing against them is embarrassing and harmful to our own social status. If a rival party has many fronts to fight on, we might expect to be attacked less, as our opponent's attention is focused on other battlefields. All these aspects play a role in how much individuals and groups engage in direct conflict, competition or more generally, situations of contest. These situations are defined by individuals exerting costly efforts, which are sunk, irrespective of victory or defeat (Konrad, 2009).

What matters in all these examples, is not primarily the level of strength, ability, social status and the number of opponents.[1] It is the asymmetry between the individuals and their opponents with respect to these characteristics. This is true on a macro-level, when countries at war need to decide on the allocation of their resources to the various conflicts they are involved in, while taking into account how many adversaries these other countries have. On the micro-level, individuals compare themselves to each other. This affects the effort exerted in competition or fights, depending on whether the opponent is perceived as strong or advantaged, or as weak or disadvantaged. When fairness is valued by the contestants, the weak increase their efforts to catch up with the strong, while the strong are more lenient in their efforts to win. How much this is the case, depends on, among other things, whether we feel empathy with others or not. This is in turn related to how socially close or distant we feel to them. If we have a strong feeling about this distance, we consider them as members of our own group or as part of a defined out-group. The latter is a question of social identity.

This thesis is concerned with how differences in strength, the number of adversaries and social preferences and identity affect individuals' behaviour in conflictive and competitive interactions. It focusses on different forms of strength, how parties are embedded in a larger system of contestants, and how they perceive each other in terms of relative material wealth and social identity. It primarily contributes to the literature on contest theory and links it to the literature on games on networks, social preference and social identity.

The first part of this thesis discusses the purely strategic interaction in bilateral conflicts, when many parties are involved. We provide the foundational model, in which such interactions can be studied under strength asymmetries. The first chapter in this part asserts that this model is 'well-behaved' in terms of its game-theoretic properties. The second chapter in this part of the thesis adds a budget constraint, to be able to study more specific problems.

[1]Throughout the thesis, the words adversary, enemy, opponent and rival are used interchangeably.

Providing readers with a tool to calculate equilibria for large sets of parameters, it provides testable hypotheses for the economic experimentation and happenstance data analysis.

The second part is focussed on how individuals' concerns about others can be modelled in the contest framework. We identify different ways to model these and list their commonalities and differences, in order to find the best trade-off between versatility and tractability of social preference models in contest. This has implications for competitive situations between individuals in the work environment, project procurement and promotions. While a lot of studies certify that differences in ability/strength can induce sabotage, it is typically the case that these differences reduce the intensity of efforts in the contest. What happens under the above concerns, which have been widely documented in the field and the lab, is unclear. Particularly when efforts are productive, this is of interest to the contest designer. Our preferred model choice also gives rise to the last project, where we use a lab experiment to induce social preferences through social identity. Whether we perceive an opponent to be socially close (in the in-group) or socially distant (in the out-group), can explain how the concerns for advantages and disadvantages arise, in more depth.

In order to outline the thesis in more detail, I want to acquaint the reader with the economic concepts of conflict and competition and why this is frequently modelled through contest. I further discuss the current state of the literature on asymmetries and conflict networks. This allows a more smooth transition to discussing systems of conflicting parties with asymmetric strength. A short introduction into social preferences and social identity prepares the reader for the second part of the thesis.

Conflict, Competition and Contest

In many classical settings in economics, resources are either distributed by a planner or an allocation mechanism, or they are produced by an individual, a group of individuals or a firm. Typically, in these settings, the total amount of resources is at least maintained if not increased.

In settings of conflict, fierce competition and contest, the involved parties exert costly effort to appropriate land, a market share or a prize of given value. In doing so they waste resources, because the costs are incurred, not only by the winner, but also by the loser (Hirshleifer, 2001). In conflict, this 'waste' of resources is directly visible through casualties and physical damage. In competition, it is rather the opportunity costs of not utilising the resources for productive means.

The contest framework of Tullock (1980) has been used across this array of applications. In this version of contest, the probability of winning, or the share of the prize is given by the ratio of a player's efforts relative to the total sum of efforts of all players. It naturally lends itself to competition for market shares and market competition, as in Nti (2000), Konrad (2000) and Kräkel and Sliwka (2006). The effort of a firm can be seen as their marketing budget, the money allocated to lobbying or as an inverse measure of prices. Studies like Szymanski (2003), Szymanski and Kesenne (2004) and Dietl et al. (2008) apply it to competition in sports. There, the term effort can be taken quite literally. The prize can either be the progression to the next round of a tournament as well as the gold medal in the Olympics. Finally, studies like Esteban and Ray (1999), Skaperdas (1998) and Abbink et al. (2010) apply theoretical contests to conflict analysis. Effort can represent physical effort in a direct fight between individuals as well as the number of soldiers sent to battle by the leaders of a country. Finally, contests are used for studies of incentives in the work environment, like Lazear and Rosen (1981), Main et al. (1993) and Harbring and Irlenbusch (2003). Further applications include rent seeking, R&D races and animal contests (Nitzan, 1994; Che and Gale, 2003; Smith, 1974).

While competition and conflict share aspects that can be modelled with contest, the terms should not be used synonymously. Competition can be fierce and a loss can be existential for an individual competitor. For example, it can mean the end of a career for a politician or a sportsman, or result in bankruptcy for a firm after an R&D race. Still, in all these cases there is hardly ever a threat of physical extinction. This can be true for violent conflict along the lines of ethnicity and religion. Examples are numerous, ranging from religious conflicts like the Hindu/Muslim relations in India and Pakistan and the Tutsi and Hutu in Rwanda to political, interstate conflicts like the two World Wars, Gulf Wars and Wars in the Middle East. In interstate conflicts, when strategists are making the deliberations, the decisions on war efforts might be representable through contest. There, the difference between contest as competition and contest as conflict within a theoretical model, is mainly a question of interpretation. When applying this to real individuals in the lab, one has to be careful. It is primarily the sunk-cost aspect of conflict that is captured with this modelling approach and the assumption that, in the lab, aggression in competition can be a proxy for aggression in conflict. Thus, in this thesis, I am widely abstracting from historic dependence, grievances or political ideology. I am concerned with the strategic aspects of contest, when applied to competition and conflict under different forms of asymmetries and preferences.

Asymmetries in Contest

In every day situations, the immediate focus of assessing possible conflict or competition outcomes is on relative strength. Football fans consider past statistics about wins and losses of their teams. In international relations, threats are made on the basis of military equipment and infantry size. The appearance of individual physical strength can deter others from picking a fight.

The absolute level of strength matters for the total damage caused or the total amount of resources wasted, even when players are identical in strength. For the decision on how much effort to exert against an opponent though, the key concept is asymmetry in strength. To model this, there are typically three variables of interest in contest: efficiencies, costs and valuations of the prize. Efficiencies modulate how efforts change the probability of winning or the share of the prize obtained. High costs reduce the range of efforts, for which a player receives a positive payoff. Valuations of the prize might differ due to the contest design or because the prize of a given monetary value is valued differently by the players. A low cost or high efficiency player can exert a higher effort for a given value of the prize. A higher valuation of winning allows a player to exert more effort while still expecting a positive net payoff. From that it should be apparent, that asymmetries in costs, efficiencies and valuations should be equivalent. This is the case in standard models, but it does not extend to network models and/or models in which preferences deviate from the standard, selfish preferences.

Asymmetries in these common knowledge primitives are well studied for the standard types of contests (Gradstein and Konrad, 1999; Nti, 1999; Baik, 1994, 2004, 2008; Epstein and Nitzan, 2002; Franke et al., 2013; Choi et al., 2016).[2]

Another source of asymmetries can be constraints on the budget used for conflict. This is realistic in military contexts when budgets are set from the government and cannot be changed on short notice. When we think of budgets not in terms of money but soldiers and equipment, that applies even more. These constraints, within the contest literature, are most common in Colonel Blotto games and crowdsourcing contests (Friedman, 1958; Feldman et al., 2005; Roberson, 2006; Kovenock and Roberson, 2012b,a; Vojnovic, 2016).

Networks and Identity Dependent Effects

Asymmetries have been studied to a great extent in two player models and the standard grand contest where n players are competing for a prize, like in a lottery. When players have a sense of who their opponents are and they are given a means of discriminating between them in terms of their actions, these effects differ.

[2] Another interesting type of asymmetries that is beyond the scope of this thesis is asymmetric information as studied in Hurley and Shogren (1998a), Wärneryd (2012) and Hurley and Shogren (1998b).

The first study that attempts to study effects along these lines is Linster (1993). There, every player has a valuation for the victory of each player in the game. It is assumed that the valuation for winning the game herself is highest. Players then have a ranking of the 'loosing outcomes' depending on which opponent wins.

The literature on networks in contest can be broadly divided into two groups: underlying networks where players share payoffs from a contest or make transfers prior or after a contest and networks on which the very contest is carried out.

In network models like Bozbay and Vesperoni (2018) and Chowdhury et al. (2012) players' gains from others are determined by an underlying network of friendships that give the players a certain share of the payoff or revenues of all players they are linked with. In the case of a combinatorial contest with k-nearest neighbours, everyone within k links from the winner also wins, while in the case of a specified contest success function for networks, each player can use the impact of other players to increase her probability of winning.

When the contest is played on the network, this can either happen in the spirit of a grand contest, where every player has to choose a single effort for all her opponents she is linked with (Rietzke and Matros, 2017; Rietzke and Roberson, 2013) or effort choices are link specific (Xu et al., 2019) and in most cases conflicts are strictly bilateral (Franke and Öztürk, 2015; Jiao et al., 2019). It is that last type of conflict networks that we consider and add to by analysing asymmetries in the priors and interpret them as measures of relative strength.

Social Preferences and a Possible Cause

While a lot of the aforementioned research focusses on asymmetries in primitives, behaviour in more general games has been shown to be affected by how asymmetric outcomes are. Social preferences as modelled in Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) have been applied to a wide variety of games including contest (Rockenbach and Waligora, 2016; Chowdhury et al., 2018; Herrmann and Orzen, 2008; Fonseca, 2009). Broadly speaking these models include a concern for disadvantageous inequality (DI), i.e., the player dislikes having a lower payoff than her opponents, and advantageous inequality (AI), i.e., they dislike to be ahead of others.

We link this to the literature on identity in economics. This has started with Akerlof and Kranton (2000) and became more prominent through Akerlof and Kranton (2010). This has been picked up in settings other than contest experimentally (Güth et al., 2008; Chen and Li, 2009; Benjamin et al., 2010) and theoretically (Bénabou and Tirole, 2011). For the contest setting Chowdhury et al. (2016) documents the result of a similar experiment as the one in Chapter 4.

This analysis can be the foundation to analyse incentives in teams. When these behavioural patterns are present in a team, increasing competitive incentives or rather forming a group identity within the organisation can influence eventual productivity and well-being.

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Part I

Contests on Networks

Chapter 1

Strength Asymmetries in Conflict Networks[1,2]

[1]This paper used to be called *Generalising Conflict Networks* (Cortes-Corrales and Gorny, 2018).

[2]We would like to thank participants from the CBESS Contest Conference 2016 and 2018 held at the University of East Anglia in Norwich and the Conflict Workshop 2018 at the University of Bath for useful comments. Also, we want to thank Subhasish M. Chowdhury, Sergio Currarini, David Hugh-Jones, Ivan Pastine, David Rojo Arjona and Mich Tvede for useful comments. All remaining errors are our own.

Sebastián Cortes-Corrales[‡] and Paul M. Gorny[§]

Abstract

When military entities are engaged in conflicts with many adversaries, they target their attacks depending on their goals in war. Understanding the interactions between these bilateral conflicts is important to assess ways to reduce conflict. We investigate the behaviour of agents in bilateral conflicts within arbitrary network structures when winning valuations and fighting technologies are asymmetric. These parameters are interpreted as measures of strength. A finite number of locally unique, pure strategy equilibria always exists. When parameters are symmetric, global uniqueness is possible. When a player starts attacking one player more strongly, others join in on fighting the victim. Different efficiencies in fighting make players fight those of similar strength. We show that asymmetry is an accelerant to conflict in *dense* networks, with many conflicts per player, and as a deterrent in sparse networks, with few conflicts per player.

Keywords: Contest, conflict, networks, games on networks

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1.1 Introduction

Competition takes the most vigorous form when the parties involved do not use resources for production or consumption, but rather to disable, destroy or appropriate resources from others (Hirshleifer, 1995; Sandler, 2000). The resources employed for these goals, in the form of soldiers, military equipment and time spent are sunk, irrespective of the final outcome. This form of competition can broadly be defined as conflict. It is this wasteful nature and the strategic aspects of conflict that spurred the interest of economists and game theorists alike.

The theoretical contributions in this domain typically model a single conflict with two or more parties. Advancements in transportation and information technology though, allow states and other international, and potentially militant interest groups to engage in multiple conflicts around the globe. That has added more complexity in how such agents are related to each other. In most of the existing models it is impossible to distinguish a fight *for* winning a single prize from a fight *against* a specific enemy. The motivation of agents is not to fight a particular opponent within the 'aggregate others', although in reality, individuals, political groups and nation states have a sense of who their opponents are. Often that plays a particular role as to why they enter the conflict in the first place.

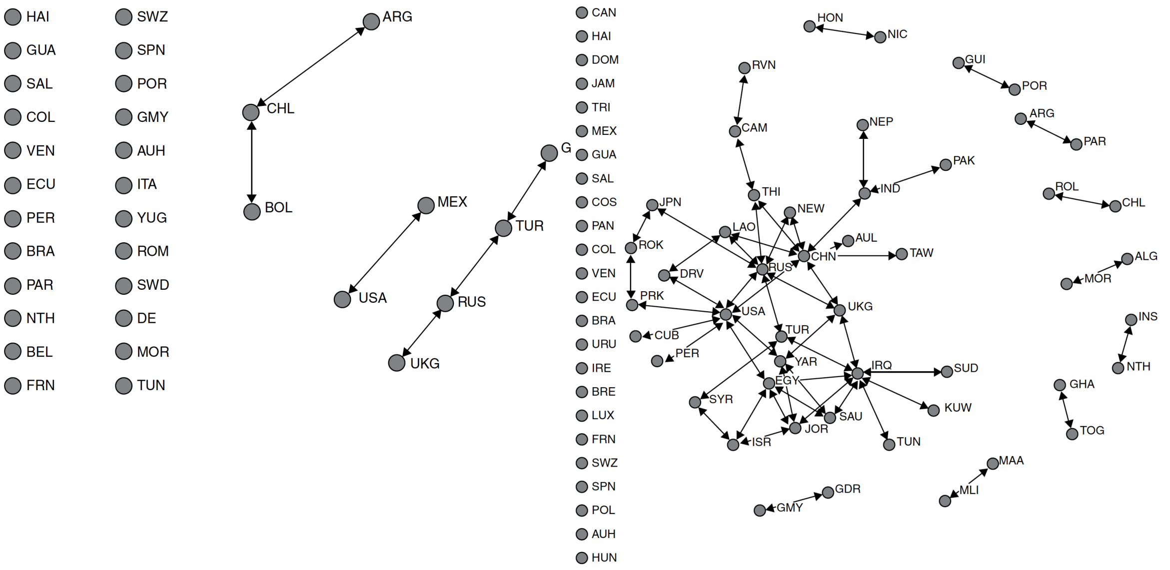
The aim of this paper is to develop a framework with multiple interconnected opponents, to understand how differences between rivals (in terms of efficiency in the conflict technology and valuation of prizes within and across bilateral conflicts) shape the optimal strategies in conflict.

Two considerations give rise to the type of model we suggest: The sunk-cost nature of conflicts and the increased complexity of violent conflicts over time.
On the one hand, conflict is characterised by sunk resource investments aiming at increasing the probability of winning a prize. This prize can be land, power or natural resources.[3] This trade-off is frequently modelled by a contest (e.g. Konrad, 2009; Vojnovic, 2016).
On the other hand, conflicts often have a structure of multiple, simultaneous conflicts between the different parties involved. Just as much as centrality, their military strength mattered when Germany engaged in battles on multiple fronts in WWI and WWII. These considerations give rise to a network of conflictual links between agents, where each link represents a bilateral contest.

To illustrate this, take a look at the set of Militarised Interstate Disputes between states[4]

[3]For applications in competition, marketing and rent-seeking that could be market share and influence in the case of marketing and lobbying, respectively

[4]A Militarised Interstate Dispute is a set of interactions involving the threat, display, or use of force between or among states (Gochman and Maoz, 1984).



(1878) (1962)

Figure 1.1: Militarised interstate disputes

Source: *Networks of Nations: The Evolution, Structure, and Impact of International Networks, 1816-2001*, (Maoz, 2010)

in 1878 – the year when the Congress of Berlin ended the Russo-Turkish War – and the type of relations in 1962 – the height of the Cold War – depicted in Figure 1.1. In both periods

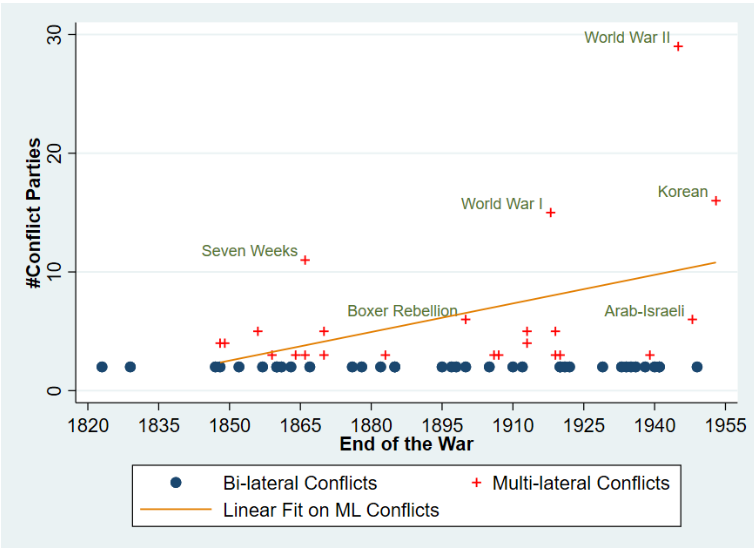


Figure 1.2: Number of Participants in Inter-State Wars

Source: Generated from the Correlates of War (COD) data set by Sarkees and Wayman (2010)

Note: We use the term 'Multi-lateral' for any conflict involving more than two parties ('Bi-lateral') here.

of time, the overall conflict structure within states is represented by networks of bilateral conflicts. In 1878, the structure of disputes was characterised mainly by networks with line components, in which a state had at most two different conflicts at the same time. In 1962 the picture is different. While there is a non-negligible number of isolated bilateral conflicts, there is a less trivial cluster of nodes centred around military powerful and/or resource rich states like the United States, Russia, China, or Iraq among others.

The left panel of the figure rather illustrates a time of peace, while 1962 was riddled with conflicts. Our claim is not that interstate wars in general became bigger and more complex (1962 is still characterised by plenty of bilateral conflicts). But the 'truly multilateral' conflicts, involving more than two parties, did. Figure 1.2 illustrates that this seems to be the case for the 19th to mid twentieth century.

It is conceptually hard to judge whether the strength of Russia made other countries engage in conflict with the US, or whether these, relatively weaker, states did so in order to oppose the threat of a potential US hegemony. In the literature on International Relations, the former is broadly comprised by the term *Bandwagoning*, while the latter is frequently referred to as *Balancing* (Waltz, 1979). The paper at hand sheds light on this question in a stylised setting.

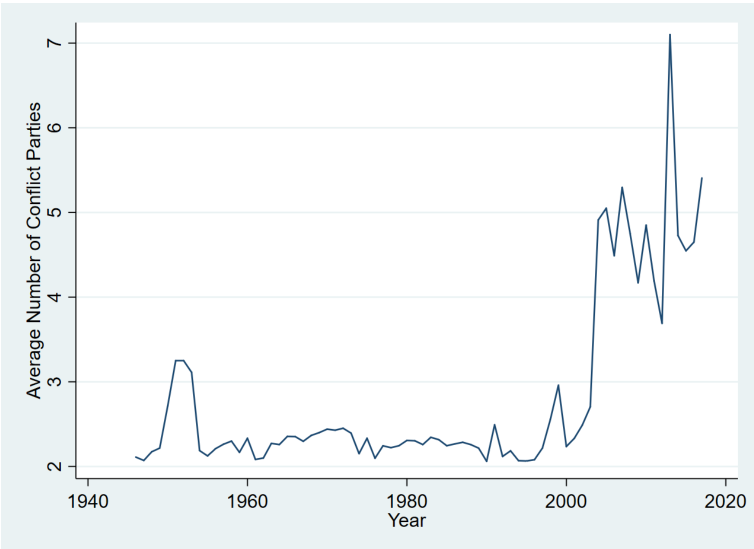


Figure 1.3: Civil Wars Involve More Parties Over Time

While the number of Militarised Interstate Disputes have declined over the second half of the twentieth century,[5] there was a sharp increase in internal and internationalised internal conflict. These types of conflict are often referred to as civil wars and include recent examples like the Syrian war, the civil war in Ukraine and the Colombian conflict. As Figure 1.3 suggests, the number of parties in military conflicts has increased on average over time. The sharpest increase happened after 9/11. Since every additional agent can have conflicts with multiple agents in the network, the increase in conflictual links is likely to be even more pronounced, giving rise to the study at hand. Note, that while every nation can just generally be more aggressive in the aggregate by sending more troops to all conflicts they are involved they can also target specific opponents. When considering these settings as grand contests, parties can only do the former. Targeting aggression towards a specific opponent rather than 'the others', is impossible. Thus, this setting calls for a model of bilateral conflicts.

We propose a setting in which there is an exogenous structure of these bilateral conflicts

[5]See Figure 1.B.1 in the appendix.

across n opponents as in Franke and Öztürk (2015) (from heron FÖ). Each link between a pair of players represents a bilateral conflict. While FÖ considers cases of symmetric characteristics, we allow for heterogeneity between opponents in terms of their efficiency of conflict investment and the valuations of winning, within and between different conflicts. We model conflict using a *contest success function* based on the axiomatisation proposed by Skaperdas (1996).

We show that a finite number of locally unique and interior Nash Equilibria exists, independently of the model parameters and under which conditions this extends to global uniqueness. In line with FÖ, we show that a general algebraic solution for the equilibrium strategies does not exist for most types of networks. We thus use implicit methods to investigate local effects of the asymmetries between the players' characteristics. We find that asymmetries in the prizes leads to Bandwagoning behaviour, mainly driven by the interaction of local network externalities induced by the conflict structure.[6] If there are players of different strength, as indicated by their efficiency to transform resources into winning probabilities, players tend to fight opponents more strongly that are similar to themselves with respect to their strength. Finally, we investigate the effects of such asymmetries on the total level of conflict intensity in the network. If one player becomes more aggressive or stronger and the network is dense (many links between the player exist), it increases. If the network is sparse, conflict reduces for an increase in one player's conflict efficiency.

Related Literature: Modelling conflict on networks is a relatively recent stream of research in economics (Dziubiński et al., 2016), starting with the model proposed by FÖ. The authors define a model, where players are embedded in a network of bilateral conflicts and each player chooses the amount of resources that they want to invest in each conflict. The conflict is modelled using a lottery contest success function. The trade-off between different conflicts is induced through a convex cost function of the total amount of resources employed. They relate the total conflict investment to the player's number of conflicts. They focus on aggregate behaviour, and thus abstract from individual characteristics by assuming symmetry with respect to all model parameters.

Beside the seminal paper by FÖ and the subsequent studies looking at this type of environment (e.g. König et al., 2017; Dziubiński et al., 2017; Matros and Rietzke, 2018)[7], there are different fields that are related to our paper. The key distinction to the afore-mentioned contributions is that the players choose an effort level for each opponent with whom they

[6]Klose and Kovenock (2015) refer to these as identity-dependent externalities. The first formalisation in contest to our knowledge can be found in Linster (1993).

[7]Huremovic (2016) studies as well conflicts on networks but with a different perspective. He is interested in the endogenous network formation of a network of conflict.

share a link, rather than choosing a single effort that they employ against all players.[8] Settings with link-specific actions, not necessarily conflict or contest though, are quite recent. To the best of our knowledge, in addition to FÖ, the only models on games on networks that introduce multidimensional strategies are Goyal et al. (2008), Bourlès et al. (2017) and Xu et al. (2019). In analysing heterogeneity of players and their response with respect to specific opponents, this characteristic is crucial. The paper that is closest to ours in nature is Xu et al. (2019). They investigate more general conflict graphs and provide expressions for comparative statics. While they are not investigating the concrete comparative statics for the case of bilateral conflicts, their approach would also lead to the problem of vanishing second order effects at symmetric equilibria.[9] We address this by considering 'discrete', arbitrarily small changes from the symmetric equilibrium to infer changes in equilibrium strategies.

Our paper is also related to the literature on multi-battle contests based on the canonical Colonel Blotto game. The Colonel Blotto game has been studied extensively since its first formulation by Borel (1921). In this game, two players simultaneously need to allocate a finite number of resources over k different conflicts. The outcome over each conflict is modelled as an all-pay-auction. This specification is well-researched with characterisations of heterogeneity between players, complementarity of prizes and other modifications to the standard formulation (Borel and Ville, 1938; Gross and Wagner, 1950; Laslier, 2002; Roberson, 2006; Hart, 2008; Hortala-Vallve and Llorente-Saguer, 2012; Weinstein, 2012; Kovenock and Roberson, 2012; Kovenock et al., 2015; Macdonell and Mastronardi, 2015; Thomas, 2018). Other studies model Colonel Blotto games, using a lottery to determine the outcome on each conflict following the Tullock (1980) contest success function. There, the probability of winning a specific conflict is a non-decreasing function of the own resource allocation and a decreasing function of the enemy's allocation. Based on this contest success function, Friedman (1958) is able to characterise the equilibrium in pure strategies of the two-player game with symmetric and asymmetric budgets and conflict valuations. The main result of that study is that the optimal allocation is proportional to the valuation of the prizes.[10] This result is a special case of our model. This strand of the literature relies on models with only two players. Our paper is a contribution to the theory of contests, in which we extend the current set of models by considering a multi-player environment with asymmetric efficiency of resources toward the conflict outcome and conflict prizes. This variation allows new insights into the effects of the interactions of local network externalities across different conflicts.

We also add to a debate in the literature of International Relations. In a war and many other

[8]Even though Dziubiński et al. (2017) studies an environment of conflicts on network with a multidimensional strategy space, this feature is due to the dynamic nature of the game.

[9]More particularly, in cases of what we will define to be a 'strictly symmetric' set of parameters.

[10]Robson (2005) generalises it by allowing the contest success function to include an effectiveness advantage and idiosyncratic noise.

conflictual settings, there is no (strong) institution that allows parties to come to a peaceful agreement. This state can be referred to as Hobbesian anarchy, due to Hobbes (1998). It is the law of the Jungle that should determine the winner(s) in such a state. Differences in the parties' strengths should thus be crucial to any analysis of conflict. Early on, Waltz (1979) and Walt (1987) coined the terms *Balancing* and *Bandwagoning*. *Balancing* is a behaviour where weaker parties ally to balance the power of a strong common opponent. *Bandwagoning* refers to the case where weak parties rally behind the strategic goals of the hegemon. There has been an ongoing discussion about which of these is more likely to occur in situations of armed conflict.[11] Our model allows to introduce a hegemon into the model, using different measures of strength, in order to shed light on the behaviour of the remaining players.

The rest of the paper is structured as follows. In the next section, we set up the model. In Section 1.3, we prove the local uniqueness and the conditions for global uniqueness of an interior Nash Equilibrium, discuss some of its properties and show the impossibility of finding a general explicit algebraic solution to the model. In Section 1.4, we study the specific family of k-regular network structures that enables us to have sharper predictions regarding the equilibrium, both in terms of individual behaviour and total conflict intensity. In this section we also present some comparative statics. Section 1.5 concludes.

1.2 The Model

Let $\mathcal{I} = \{1, \ldots, n\}$ be a finite set of players with $n \geq 2$. All conflicts are contained in $B \subseteq \mathcal{I}^2$ where $\mathcal{I}^2$ is the set of unordered pairs of $\mathcal{I}$ with typical element (ij). The underlying conflict network $\mathcal{G}$ is represented by the connected graph associated with the pair of sets $(\mathcal{I}, B)$.[12] We say that any pair of players i and j is involved in a bilateral conflict (ij) if and only if $(ij) \in B$. It appears to be natural that a conflict does not disappear if it is ignored. Also, no player can be the enemy of herself. Thus, we assume the network $\mathcal{G}$ to be *undirected* ($\forall i, j \in \mathcal{I} : (ij) \in B \Leftrightarrow (ji) \in B$) and *irreflexive* ($\forall i \in \mathcal{I} : (ii) \notin B$). Let $N_i = \{j \in \mathcal{I} | (ij) \in B\}$ denote the set of i's rivals. The total number of i's rivals is given by $d_i = |N_i|$. The total number of conflicts is $b = \frac{1}{2}\sum_i d_i$. We denote $S \subseteq \mathcal{I}$ a clique of $\mathcal{G}$, if every pair of players i and j in S has a conflict between them – i.e., $(ij) \in B$. In each bilateral conflict, $(ij) \in B$, players i and j fight for a strictly positive, exogenous prize. Player i's valuation of winning the prize against player j is denoted $v_{ij} > 0$. This framework can accommodate constant-sum bilateral conflicts, when $v_{ij} = v_{ji}$, or non constant-sum bilateral conflicts, when $v_{ij} \neq v_{ji}$.[13]

[11] See for example Schweller (1994), Waltz (1997) and Lieber and Alexander (2005).

[12] A graph $\mathcal{G}$ is connected if for every pair of players i and j in $\mathcal{I}$ we can find a sequence of adjacent conflicts to 'travel' from i to j. The results that we are presenting hold for any non-trivial component of any disconnected network. Therefore, we do not consider such network structures in our analysis.

[13] The latter case can also capture the idea of identity externalities as mentioned in the introduction.

Each player i can exert effort $x_{ij} \in \mathbb{R}_+$ to increase her probability of winning the conflict against player j. We denote player i's action by $\mathbf{x_i} = (x_{ij})_{j \in N_i}$, which is a d_i-dimensional vector that contains all her effort choices.

The outcome of each bilateral conflict is determined by the total amount of efforts spent on that specific conflict. Player i's probability of winning is determined by a *contest success function* (from hereon CSF) $p(a_i x_{ij}, a_j x_{ji})$, where $a_i \geq 1$ captures how efficiently player i can employ her resources to increase this probability.[14] The CSF is increasing and concave in x_{ij} and decreasing and convex in x_{ji}. Further, it does not depend on any x_{lk} with $(lk) \neq (ij)$. The axiomatised class of CSFs by Skaperdas (1996) satisfies these properties. Thus, the probability of i, winning the prize in the conflict against j, obtains as

$$p_{ij} = p(a_i x_{ij}, a_j x_{ji}) = \begin{cases} \frac{f(a_i x_{ij})}{f(a_i x_{ij}) + f(a_j x_{ji})} & \text{if } (x_{ij} + x_{ji}) \neq 0 \\ \frac{1}{2} & \text{if } (x_{ij} + x_{ji}) = 0, \end{cases} \tag{CSF}$$

where a random draw with equal probabilities determines the winner if both players exert zero effort. The impact function $f(.)$ is a positive and strictly increasing function of its argument with $f(0) = 0$ and is at least twice differentiable. To meet the aforementioned concavity of the CSF, we are consequently assuming $f''(a_i x_{ij})(f(a_i x_{ij}) + f(a_j x_{ji})) - 2f'(a_i x_{ij})^2 < 0$. For ease of notation, throughout the rest of the paper, let $\boldsymbol{\omega} = (\boldsymbol{v}, \boldsymbol{a})$ be the combination of b values collected in $\boldsymbol{v}$ and the n efficiencies collected in $\boldsymbol{a}$. The space of all such combinations is $\Omega \subseteq \mathbb{R}^{b+n}_{++}$. We distinguish between *symmetric* and *strictly symmetric* parameterisations. A *symmetric* parametrisation is any $\boldsymbol{\omega} \in \Omega$ such that $v_{ij} = v_{ji}$, without a restriction on $\boldsymbol{a}$. We call an arbitrary case where all valuations across players and conflicts, and all efficiencies across all players are the same, a *strictly symmetric* parameterisation and denote it $\overline{\boldsymbol{\omega}} \in \Omega$.[15]

All players face the same cost function. The costs of exerting effort are captured by $C(X_i)$, where $X_i = \sum_{j \in N_i} x_{ij}$ denotes the total amount of effort spent by player i across all her conflicts. We assume the cost function to be at least twice differentiable, strictly convex and use $C'(0) = 0$, making it strictly increasing for every $X_i > 0$.[16] The slope and curvature of $C(X_i)$ determine the magnitude of opportunity costs for player i of exerting effort across conflicts. If the function is strongly increasing, player i has to withdraw resources from other conflicts, rather than increasing the total amount of efforts spent.

For each conflict $(ij) \in B$, agent i's expected revenue is $\pi_{ij} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ such that $\pi_{ij} = p_{ij} v_{ij}$. We assume that agents are expected payoff maximisers with risk-neutral preferences.

[14] Think of this as the ratio of efficiencies relative to the weakest player w, given by $\frac{a_i}{a_w} \geq 1$. This assumption avoids problems with typical cost functions later. For most of our results it is not essential.

[15] This is a slight abuse of notation. In fact $\overline{\omega}$ is the whole set $\{(\lambda_1 \mathbf{1}_b, \lambda_2 \mathbf{1}_n) | (\lambda_1, \lambda_2) \in \mathbb{R}^2_{++}\}$, where $\mathbf{1}_k$ is a $k \times 1$ vector containing only ones. Our results hold for any element of this set.

[16] In fact our results also go through for sufficiently small $C'(0) > 0$.

We consider an additively separable payoff function, given by

$$\Pi_i(\mathbf{x_i}, \mathbf{x_{-i}}, \mathcal{G}) = \sum_{j \in N_i} \pi_{ij} - C(X_i).$$

The set of players, the network structure, the action spaces and the expected payoffs define a simultaneous game under complete information $\Gamma = (\mathcal{I}, \mathcal{G}, \mathbb{R}_+^{2b}, (\Pi_i)_{i \in \mathcal{I}})$. Our objective is to study the properties and changes in the Nash equilibria of this game, focussing on changes in efficiencies and valuations of prizes. We thus first show the existence of such equilibria in a more general setting, in order to analyse the effects of the afore-mentioned asymmetries on individual and total efforts.

1.3 Equilibrium Analysis

Given the above structure, each player faces the following maximisation problem of dimension d_i determined by the network structure $\mathcal{G}$ for a given $\mathbf{x_{-i}}$,

$$\max_{\mathbf{x_i} \in \mathbb{R}_+^{d_i}} \Pi_i(\mathbf{x_i}, \mathbf{x_{-i}}, \mathcal{G}). \tag{1.1}$$

For every player $i \in \mathcal{I}$, the equilibrium behaviour is described by the typical balance of marginal benefits and marginal costs for each conflict $(ij) \in B$.

$$\frac{a_i f'(a_i x_{ij}) f(a_j x_{ji})}{(f(a_i x_{ij}) + f(a_j x_{ji}))^2} v_{ij} = C'(X_i). \tag{PCC}$$

The game is a continuous game[17] with compact strategy spaces[18] and a finite set of players. Thus, we can apply the theorems due to Debreu (1952), Glicksberg (1952) and Fan (1952) in order to guarantee the existence of a pure-strategy Nash equilibrium.[19] Due to the continuity of the cost function and the arbitrarily large marginal gains close to $(0, 0)$ on each conflict, equilibria are strictly interior. If that is given, the system of first order conditions (from hereon referred to as F) characterises these equilibria. We show that the determinant of that system is always strictly larger than zero. Together with the uniqueness result akin to FÖ, this provides us with the following result.

[17] Technically we are using the contest success function

$$p(a_i x_{ij}, a_j x_{ji}) = \frac{f(a_i x_{ij}) + \delta}{f(a_i x_{ij}) + f(a_j x_{ji}) + 2\delta}$$

for some arbitrarily small $\delta > 0$. This approach is essentially the one suggested in Myerson and Wärneryd (2006) and has also been used by FÖ.

[18] We can define some arbitrarily large but finite $M_i > 0$ such that for each $i \in \mathcal{I}$ and each $(ij) \in B$ we have $x_{ij} \in [0, M_i]$.

[19] All of these papers rely on Kakutani's fixed point theorem. Since the structure of our game results in unique best responses for any given set of parameters and admissible $f(\cdot)$ and $C(\cdot)$, a proof using Brouwer's fixed point theorem would be sufficient.

Proposition 1.1 (Existence, (local) Uniqueness and Interiority of Pure Strategies)**.**
A finite number of locally unique, interior, pure-strategy Nash equilibria exist $\forall \boldsymbol{\omega} \in \Omega$*. The solution function* $x(\boldsymbol{\omega}) : U \subseteq \Omega \mapsto X \subseteq \mathbb{R}^{2b}_{++}$*, mapping any parameter* $\boldsymbol{\omega}$ *into a Nash equilibrium* $x(\boldsymbol{\omega})$*, is at least* C^2 *and its derivative is given by*

$$D_x(\boldsymbol{\omega}) = -\left[D_x F(x(\boldsymbol{\omega}); \boldsymbol{\omega})\right]^{-1} D_{\boldsymbol{\omega}} F(x(\boldsymbol{\omega}); \boldsymbol{\omega}) \tag{1.2}$$

Further, there exists an open neighbourhood around any symmetric parametrisation, such that the equilibrium is globally unique.

The convex cost function creates a trade-off between any two effort levels exerted by some player i. The CSF creates a relationship between effort levels across connected players. Solving the set of first order conditions becomes a recursive problem. The following result formalises that observation.

Proposition 1.2.
There exists an indirect global dependence where for any pair of agents h *and* l *who are not rivals (i.e.* $h \notin N_l$ *and* $l \notin N_h$*), the effort levels as characterised by the system of first order conditions* F *can implicitly be expressed as* $\mathbf{x}_h\left(\mathbf{x}_l, \mathbf{x}_{-\{h,l\}}\right)$ *and* $\mathbf{x}_l\left(\mathbf{x}_h, \mathbf{x}_{-\{l,h\}}\right)$*.*

In that sense, the fact that we have a connected network creates indirect relations between all players throughout the rivals of rivals along any path of the network. While this does not mean that players incorporate distant rivals' actions directly in their maximisation problem, the equilibrium behaviour reflects this indirect connection. We use this to show that an algebraic solution for the equilibria can be obtained for hardly any network structure. How much any player's actions affect another player's effort levels in equilibrium depends on how long the shortest possible path between them is. Consider for example a line network with 6 players as in Figure 1.4. Based on Proposition 1.2, we can mathematically express any of her best responses as $x_{12} = x_{12}(x_{23}(x_{34}(x_{45}(x_{56}))))$.

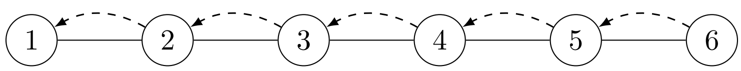


Figure 1.4: Line Network with 6 Players

Notice, that through the use of equation (PCC), each of these nested functions applies a square to a sum. To find the equilibrium strategies, we require to find the roots of at least one general polynomial of degree 2^5.
Denoting the length of the longest path in a given network with L,[20] solving the system of first order conditions for all players, generally requires us to find the roots of at least one general polynomial of degree no less than 2^L. This is a mathematical impossibility for any L greater than 2, according to the Abel-Ruffini Theorem (1779).

[20] Remember, that we are considering all shortest paths between players first. 'Longest' then refers to the longest among these shortest paths.

Corollary 1.1.
The equilibrium strategies of the game do not have a generic algebraic solution if the length of the longest path between any two players is greater than or equal to 3.

By a generic algebraic solution we mean any formula which would express the roots of the polynomial as functions of the model parameters by means of algebraic operations (i.e, $+$,$-$,$\times$ or /) and roots of natural degrees. We can still gain further insights into equilibrium behaviour by imposing some degree of symmetry on the network structure.

1.4 k-Regular Networks

Given any $\boldsymbol{\omega} \in \Omega$, Proposition 1.1 provides the general form of the matrix of derivatives for any equilibrium. If we focus on a subset of network structures, it is possible to obtain closed forms of these matrices to assess comparative statics more precisely. As the following expressions appear more often in the subsequent part of the paper, denote the derivative of p_{ij} with respect to its first argument by $p^1_{ij} = p^1(a_i x_{ij}, a_j x_{ji})$ and the derivative with respect to the second argument by $p^2_{ij} = p^2(a_i x_{ij}, a_j x_{ji})$.[21] The second and cross-derivatives are given by $p^{11}_{ij} = p^{11}(a_i x_{ij}, a_j x_{ji}) = -p^{22}_{ji}$ and $p^{12}_{ij} = p^{12}(a_i x_{ij}, a_j x_{ji}) = -p^{21}_{ji}$, respectively. The set of graphs we consider is defined as follows.

Definition.
A k-regular network is any graph, for which $d_i = k$ *for all* $i \in \mathcal{I}$ *and some* $k \in \{1, ..., n-1\}$.

This family of graphs includes the complete network ($k = n-1$) and the ring (or minimal connected) network ($k = 2$), as well as some networks in between these extreme cases. Within these networks, it is possible to characterise the equilibrium strategies in case of a strictly symmetric parametrisation as in FÖ.
The network structure in that class of graphs has a clear relationship with the equilibrium efforts exerted. In this set of cases we are able to infer the exact network structure from the strategies played by strictly symmetric players.

Proposition 1.3.
For any $\overline{\boldsymbol{\omega}}$*, the conflict network is k-regular if and only if the equilibrium for all* $(ij) \in B$ *is*

$$x_{ij} = x^s = \frac{1}{k} C'^{-1}\left(\frac{f'(\overline{a}x^s)}{4f(\overline{a}x^s)}\overline{av}\right) > 0. \tag{1.3}$$

[21] Thus, the derivative of p_{ij} with respect to x_{ij} is given by $p^1_{ij}\dfrac{\partial(a_i x_{ij})}{\partial x_{ij}} = a_i p^1_{ij}$.

In real world conflicts, where effort levels are more measurable than other parameters of the model, this result allows to infer the underlying network structure. From now on, we denote $x(\boldsymbol{\omega})$ the Nash equilibrium at some $\boldsymbol{\omega} \in \Omega$, with typical element $x_{ij}(\boldsymbol{\omega})$ for conflict (ij) and player i. We use the shorthand $x^s = x(\overline{\boldsymbol{\omega}})$ for an arbitrary strictly symmetric equilibrium in a k-regular network.

In such an equilibrium, the first order conditions are the same for every player since $X_i = X = kx^s$. We arrive at the same comparative statics as FÖ for the parameters they share with our model.

$$\frac{\partial x^s}{\partial v} > 0, \quad \frac{\Delta x^s}{\Delta k} < 0$$

The effect of efficiencies on effort levels is not clear in general. There exists a trade-off between increasing the probabilities of winning, while keeping costs constant and reducing costs while keeping probabilities constant. The overall effect depends on how sensitive the probability of winning is to changes in the ratio of impacts.[22] How the CSF behaves, depends crucially on the properties of the impact function.

Canonical Case (FÖ).

In a canonical setting, with the widely used functional form of the impact function $f(a_i x_{ij}) = (a_i x_{ij})^r$ *for sensitivity parameter* $r \in (0, 2)$[23] *and cost function* $C(X_i) = \frac{1}{\rho} X_i^\rho$ *for* $\rho > 1$, *the effects cancel out. The impact function used here, is such that the CSF becomes homogenous of degree zero. Thus, multiplying both effort levels in a bilateral contest by the same factor, the probabilities remain the same.*[24]

In this setting, the equilibrium as defined in Proposition 1.3 is given by

$$x^s = \left(\frac{1}{k}\right)^{\frac{\rho-1}{\rho}} \left(\frac{r\overline{v}}{4}\right)^{\frac{1}{\rho}}.$$

The equilibrium corresponds to the one presented by FÖ for $r = 1$ *and* $\rho = 2$. *It is clear that if we increase the number of conflicts per players in that network* (k), *the equilibrium strategies are going to decrease at the rate of* $\sqrt{k}$. *This is lower than the rate at which conflicts increase. This means that total efforts increase.*

Turning to changes in efficiencies, we see that $\frac{\partial x^s}{\partial \overline{a}} = 0$. *Due to homogenous impact functions the marginal probabilities cannot change when moving from one symmetric equilibrium to another. Thus, the level of marginal costs, and with it the level of total and individual efforts, remain constant. Inspection of expression* (1.3) *shows that for non-homogenous impact functions, a change in total efficiencies might have an effect on symmetric efforts.*

[22] The CSF can, apart from $(0, 0)$, alternatively be written as $\frac{1}{1+\frac{f(a_j x_{ji})}{f(a_i x_{ij})}}$.

[23] See e.g. Pérez-Castrillo and Verdier (1992) applied to bilateral contest for this restriction.

[24] FÖ obtains for $r = 1$ and $\rho = 2$.

Note that for close to linear costs (ρ close to unity) and $r = 1$, every player is exerting the two player Nash equilibrium effort $\frac{1}{4}v$ in each conflict. This is intuitive, as with a linear cost function, there are no opportunity costs between conflicts.

We stop investigating symmetric changes of parameters here, as this paper is concerned with asymmetries in conflict networks. In Proposition 1.1, we already gave an abstract characterisation of the derivatives at any given equilibrium. For k-regular networks, we can derive comparative statics for any strictly symmetric equilibrium, for which we can determine the signs and magnitudes.

Proposition 1.4.
In a k-regular network the partial derivatives around the equilibrium at an arbitrary strictly symmetric parametrisation $\overline{\boldsymbol{\omega}} \in \Omega$ can be obtained analytically as

$$\begin{aligned}
\frac{\partial x_{ij}}{\partial v_{ij}} &= -\frac{z-(k-1)C''(X)}{z-kC''(X)}\frac{\overline{a}p^1}{z} > 0 \\
\frac{\partial x_{il}}{\partial v_{ij}} &= -\frac{C''(X)}{z-kC''(X)}\frac{\overline{a}p^1}{z} < 0 \quad \textit{for all } l \neq j \\
\frac{\partial x_{ij}}{\partial a_i} &= -\frac{1+z}{z-kC''(X)}\left(\frac{p^1\overline{v}}{z}+\frac{x^s}{\overline{a}}\right) \lesseqgtr 0,
\end{aligned} \tag{1.4}$$

for all $i \in \mathcal{I}$ and $(ij) \in B$, where $z = \overline{a}^2 p^{11}\overline{v}$. All other partial derivatives vanish.

We observe that for changes in valuations, the trade-off between conflicts is mediated by the convexity of $C(\cdot)$. If $C''(X) \to 0$, player i only increases her effort on (ij) without reducing it anywhere else.[25]

It is interesting to note that the sign of the effect of an increase in a_i is ambiguous. This is different from our earlier result, as the change in efficiencies considered here is asymmetric. Only player i's efficiency increases, while keeping all other players' efficiencies constant. This way, i becomes a strong player relative to the remaining players in the network.

Remember that a change in efficiency creates a trade off for player i. She can increase her probabilities of winning, while keeping costs constant. Alternatively, she can keep probabilities of winning constant, while reducing her total costs. To see this more clearly, consider her payoff when all her opponents play the symmetric effort choice.

$$kp(a_i x_i(k), \overline{a}x^s(k))\overline{v} - C(kx_i(k)) \tag{1.5}$$

Since a_i is player-specific, but does not vary across player i's conflicts, we can write her efforts as x_i for all opponents $j \in N_i$. Also keep in mind that x_i is an infinitesimal change *relative* to x^s which depends on k.

[25] We discuss $C''(X) \to \infty$ in the next chapter.

The number of conflicts (k) affects the number of prizes that player i can win, the amount of symmetric equilibrium effort and the amount of total effort, given a choice of x_i. The individual efficiency increases player i's impact for a given level of effort in all her conflicts. The above mentioned trade off becomes apparent by comparing the potential marginal gains and marginal costs of an increase in a_i. For the canonical choice of functional forms these always cancel out.

Proposition 1.5.

Let $f(a_i x_{ij}) = (a_i x_{ij})^r$ for $r \in (0,2)$ and $C(X_i) = \frac{1}{\rho} X_i^\rho$. At any strictly symmetric equilibrium we have

$$\frac{\partial x_{ij}}{\partial a_i} = 0.$$

This result is mainly due to the fact that the impact function is homogenous (of degree r) in its argument $a_i x_{ij}$. This results in the highest marginal utility to occur at $a_i x_{ij} = a_j x_{ji}$. Given that $x_{ij} = x_{ji} = x^s$, this implies that the maximum occurs at $a_i = a_j = \overline{a}$. For an infinitesimal change of a_i, the effect is thus zero.[26] Non-homogenous functions can still result in effects that are different from zero.[27]

The following result is immediate, but important for large parts of the remaining analysis.

Corollary 1.2.

For the canonical functional forms, at any strictly symmetric equilibrium we have

$$\frac{\partial (a_i x_{ij})}{\partial a_i} > 0. \tag{1.6}$$

While the amount of soldiers sent to a conflict does not change when given better technology, one could say that the *efficiency units* of soldiers increases.

This allows to investigate novel interaction effects across the network, presented in the following sections.

1.4.1 Individual Efforts

Interestingly, the effect any change in efficiency has on the behaviour of the remaining players in the network, seems to be independent of the sign of Δa_i. This is due to the fact that

[26] If the marginal probability was given by a function $g(a_i x_{ij})$, its graph w.r.t. a_i would have an inverted U-shape. Thus, any discrete change would have a negative effect on x_{ij}.

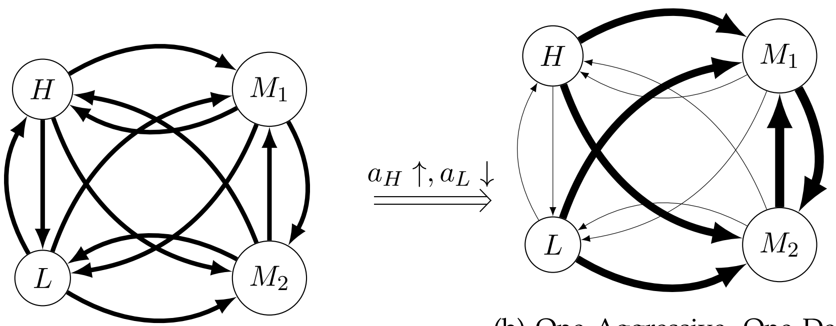
[27] Note that this is not just a specific case of a homogenous CSF. Given the previous axioms, this is the only functional form that additionally also allows for homogeneity (Skaperdas, 1996).

the slope of the best-response function in each conflict, implicitly characterised by the FOC, always has the same sign as the cross-derivative of the CSF. We have $p_{ij}^{12} = p_{ji}^{21}$ whenever $a_i x_{ij} = a_j x_{ji}$, while $p_{ij}^{12} = -p_{ji}^{21}$ always holds as probabilities of winning are exclusive. Thus, we know that $p_{ij}^{12} = p_{ji}^{21} = 0$ for $a_i x_{ij} = a_j x_{ji}$. Since one can show that this is a maximum of $x_{ij}(x_{ji})$ with respect to x_{ji}, player i reduces her effort if j changes her effort in either direction.

Proposition 1.6.
Fix a k-regular network and some $i \in S$. *Let* $\boldsymbol{\omega}' = (\overline{v}, ..., \overline{v}, \overline{a}, ..., a_i = \overline{a} + \epsilon, ..., \overline{a})$. *Furthermore, let* $\Delta x_{lq} = x_{lq}(\boldsymbol{\omega}') - x^s$ *for any* $l, q \in S$. *There exists some* $\epsilon^* > 0$ *such that for* $\epsilon \in (0, \epsilon^*)$ *the equilibrium* $x(\boldsymbol{\omega}')$ *for all* $j, m \in S \backslash \{i\}$ *satisfies*

$$\Delta\left(a_i x_{ij}\right) > \Delta x_{jm} > \Delta x_{ij} = 0 > \Delta x_{ji}$$

Irrespective of whether player i increases or reduces her efforts following a change in a_i, the other players in the network reduce their efforts towards her.



(a) Symmetric Case

(b) One Aggressive, One Defensive Player

Figure 1.5: Diagrammatic Representation of Proposition 1.6 for $k = 4$
Note: The size of the arrows in the two panels refers to the relative amount of efforts exerted in each bilateral conflict.

Figure 1.5 illustrates a case of four players (the nodes) in a complete network that represents a clique S. The magnitude of the arrows between nodes indicates the relative level of efforts within each player's conflicts.

Panel (a) represents the symmetric case, in which all efforts are the same (x^s). Assume now that player H's efficiency increases, while player L's efficiency reduces. We see in panel (b), that players M_1 and M_2 concentrate their efforts against each other. Since their reduction of effort is an effect of the efficiency change in H and L, it is smaller than the initial change in $a_H x_{Hj}$ and $a_L x_{Lj}$ ($j \in \{L, H\}$). Thus, players H and L divert their efforts from their common conflict towards players M_1 and M_2.[28]

[28] This change of two parameters at a time is a straightforward variation of Proposition 1.6.

Given a specific level of costs that is prohibitive, efficiencies become a scaling factor of effective budgets. Since, after applying the change, we have $a_H > a_L$, one can interpret the above figure in terms of the fight between players with unequal endowment. The prediction of the model at hand is that conflict intensity contracts towards the mediocrely endowed individuals and away from the rich and the poor.
If efficiency is a measure of strength, and we either delete player L, we are in a situation where a single player, H in this case, is stronger than the rest. The model suggests that the weaker players rather fight each other. While Bandwagoning typically needs the weak to rally behind the strong player, who in turn fights them less/ceases to fight them, Balancing (i.e., the weak teaming up to oppose the strong) is not the strategically optimal behaviour.

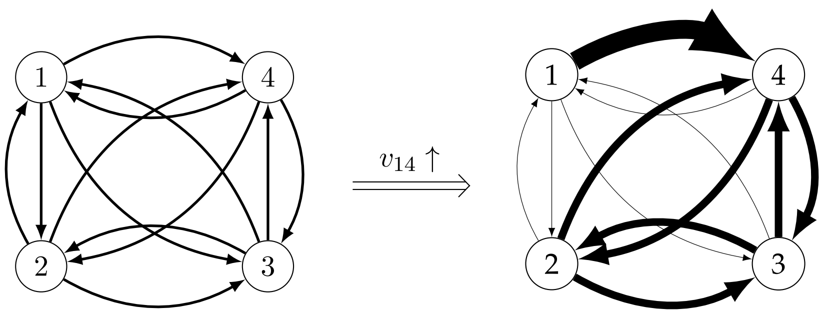
Any change in efficiencies bears a certain degree of symmetry with it, since Δa_i affects all conflicts of player i equally. A change in valuations induces more asymmetric strategic responses.

Proposition 1.7.
Fix a k-regular network and some $i, j \in S$. Let $\boldsymbol{\omega}' = (\overline{v}, ..., v_{ij} = \overline{v} + \epsilon, ..., \overline{v}, \overline{a}, ..., \overline{a})$. Furthermore, let $\Delta x_{lq} = x_{lq}(\boldsymbol{\omega}') - x^s$ for any $l, q \in S$. There exists some $\epsilon > 0$ such that the equilibrium $x(\boldsymbol{\omega}')$ for all $h, m \in S \backslash \{i, j\}$ satisfies

$$\Delta x_{ij} > \Delta x_{jh}, \Delta x_{hj}, \Delta x_{hm} > 0 > \Delta x_{ji}, \Delta x_{hi} > \Delta x_{ih} \tag{1.7}$$

Figure 1.6 exemplifies this statement for the case of four players. The increase in effort, following an increase in player 1's value v_{14}, is intuitive. The effects on the other players' efforts are less obvious. The effect of increasing v_{14} has a first-order effect only on player 1's effort levels and a second-order effect on the remaining players. This is due to how the values feed into the players' payoffs. While each own valuation has a direct effect on her payoff, it can only affect other players' payoffs through the strategic channel. Additionally, the symmetric equilibrium is characterised through $p^{12}(\overline{a}x^s, \overline{a}x^s) = 0$, so the strategic effects do not show up immediately. That is, the fact that even 'impartial' players change their equilibrium effort in case of a change in some other player's values is a result of the interdependencies of conflicts.

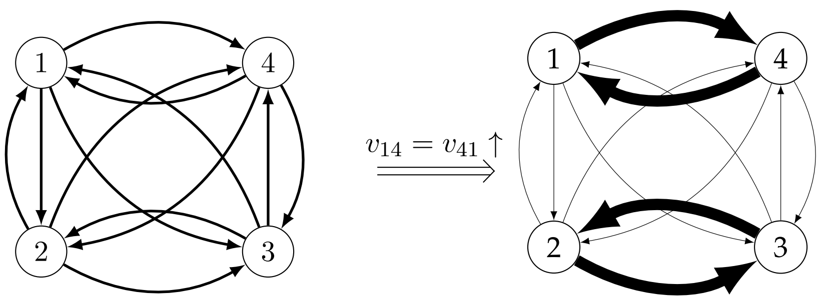
The behaviour here is an even clearer prediction of Bandwagoning as opposed to Balancing. Player 1 attacks player 4 more aggressively and fights players 2 and 3 less. This reduction in effort against them, leads players 2 and 3 to reduce their efforts against player 1 as the marginal revenue for these conflicts decreases. The same is true for player 4 due to the increased effort against her. As in the earlier result, this increases infighting among



(a) Symmetric Case (b) Asymmetric Case

Figure 1.6: Diagrammatic Representation of Proposition 1.7 for $k = 4$

players 2,3 and 4.
Interpreting a high valuation as strength is a common interpretation in the contest literature.[29] This result could equally well be described as a form of bullying, where one individual decides to bully a peer and so-called bystanders follow the bully and become hacks. The hacks are also fighting against each other, but they do not have to fear fierce attacks from the bully. The fact that significant shares of adolescents are observed to behave in that matter is well-established in social psychology (see for example Craig and Pepler (1998), O'connell et al. (1999) and Salmivalli et al. (1996)).



(a) Symmetric Case (b) Asymmetric Case

Figure 1.7: A Conflict Becomes More Valuable

In the special case where two players increase their valuation of winning against each other in the same way ($v_{ij} = v_{ji}$), is considered in Figure 1.7. In this case, players 1 and 4 fight each other more fiercely, due to an increase in $v_{14} = v_{41}$. This is expected. Their reduction in efforts against players 2 and 3 results in infighting among those players. The figure thus suggests that whenever two parties quarrel, the third parties quarrel as well.

Arrows in the above graphs compared effort levels to the symmetric equilibrium and in some cases to the other efforts of the same player. How changes in effort levels compare across players depends on the size and density of the network, as well as on other parameters of the model. We now turn to settings of these primitives, where such a comparison is possible for asymmetric parameters, to analyse conflict intensity over the entire network.

[29]This interpretation as strength or ability comes from the fact that a shift in valuation is isomorphic to a shift in costs, since a contestant with payoff $p(x_i, x_j)v_i b - x_i$ shows the same behaviour as one with payoff $p(x_i, x_j)v_i - \frac{1}{b}x_i$.

1.4.2 Total Efforts

With the insights into how individual efforts change when parameters are asymmetric, we are ready to investigate an aggregate measure of conflict intensity. Let $\mathcal{X} = \sum_{i \in \mathcal{I}} X_i$ denote total effort spent by all players in the network, across all their conflicts. Let $T(k) = k - 2 + \frac{p^{11}}{C''(X)}$ with $X = kx^s$. This term relates the concavity of the maximisation problem to the density of the conflict network, in which players are embedded.

Proposition 1.8.

Fix a k-regular network and some $i \in \mathcal{I}$ and some small $\Delta a_i = a_i - \overline{a} > 0$.We have

$$\frac{\Delta \mathcal{X}^s}{\Delta a_i} \begin{cases} > 0 & \textit{if} \quad T(k) > 0 \\ = 0 & \textit{if} \quad T(k) = 0 \\ < 0 & \textit{if} \quad T(k) < 0 \end{cases}$$

The effects of a change in v_{ij} are simpler, as the individual effect is clearly positive.

Proposition 1.9.

Fix a k-regular network and some $i, j \in \mathcal{I}$ and some small $\Delta v_{ij} = v_{ij} - \overline{v} > 0$. For $T(k) > 0$, we have $\frac{\Delta \mathcal{X}}{\Delta v_{ij}} > 0$.

It is not possible to clearly determine cases where total efforts decrease. When $T(k) < 0$ effects for player i and the remaining players go in opposite directions and their magnitude depends again on Δv_{ij}.

In conventional models of multiplayer contest, it has been shown that asymmetry in efficiencies decreases efforts. Consider a grand contest with n players and linear costs where $n - 1$ players have efficiency 1 and player i has efficiency a_i. This setting is the closest standard contest model to our setting.[30] The equilibrium strategies are given by

$$x_i = \frac{a_i(n-1)^2 - (n-1)(n-2)}{a_i^2 n^2} v \qquad \text{and} \qquad x_j = x = \frac{n-1}{a_i n^2} v \quad \text{for all } j \neq i.$$

The total effort with asymmetric efficiencies is thus given by

$$\mathcal{X} = x_i + (n-1)x = \frac{2a_i(n-1)^2 - (n-1)(n-2)}{a_i^2 n^2} v$$

One can see that this expression tends to zero as asymmetry ($\frac{a_i}{1} > 1$) becomes arbitrarily large. In fact, the derivative at the symmetric equilibrium ($a_i = 1$) is given by $-2\frac{n-1}{n^2}$. In the canonical setting we see that players in conflict networks do not necessarily behave in that

[30] Since we consider a strictly convex cost function, this comparison only holds for arbitrarily low cost convexity. For example ρ close to 1.

way, when the network is *dense*, that is, when for a given number of players, the number of links k is large.

Canonical Case (FÖ).

In this case we have $T(k) > 0$ whenever

$$k > \overline{k} = \frac{2(\rho - 1)\overline{av}}{(\rho - 1)\overline{av} - 1}$$

From Proposition 1.8 we know that in this case we have $\frac{\Delta \mathcal{X}}{\Delta a_i} > 0$. To compare this with the grand contest with $r = 1$ and $\overline{a} = 1$ from before, we choose ρ close to 1 to approximate linear costs. Thus, for this case we have $\frac{\Delta \mathcal{X}}{\Delta a_i} > 0$ if $k > 0$, which is trivially fulfilled. In such a conflict network, asymmetry has the opposite effect on total efforts as is thus far documented in the contest literature.

The intuition is as follows. As player i changes her efforts away from the symmetric equilibrium, there is more infighting in the rest of the network. This can be seen again by looking at figure 1.5, considering the cases where player L is excluded. Due to player H's increased effective effort, players M_1 and M_2 increase their efforts against each other. If a lot of these players M exist, this increase in infighting will be more pronounced than the decrease of efforts against player H.

For $\frac{\Delta \mathcal{X}}{\Delta v_{ij}} > 0$, the intuition is the same. A dense network allows a lot of opportunities to increase efforts against all players other than i, and thus any other effect becomes relatively small as k increases.

We can characterise changes in total effort, conditional on restrictions on k, and are able to investigate a specific canonical case that compares well with the grand contest.

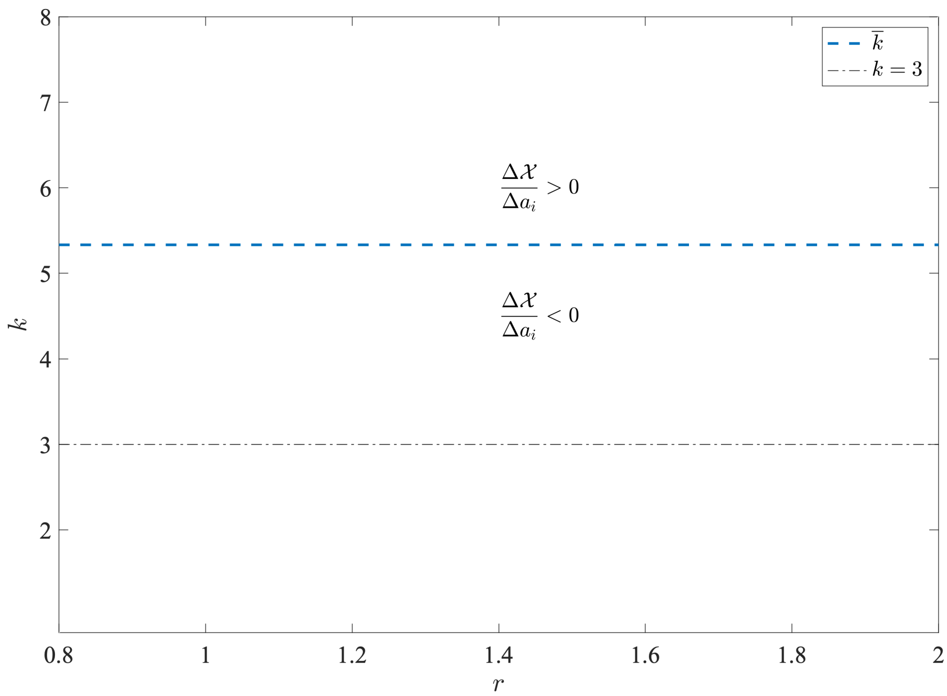


Figure 1.8: Different Regions for k According to Proposition 1.8.
Note: This graph is obtained for $\overline{a} = 1$, $\rho = 1.2$ and $\overline{v} = 8$.

First, note that the lowest integer for which $T(k) > 0$ is possible is 3. Above this level, we have $k < \overline{k}$ and thus $\frac{\Delta\mathcal{X}}{\Delta a_i} < 0$, while for $k > \overline{k}$, we have $\frac{\Delta\mathcal{X}}{\Delta a_i} > 0$. On the dashed line, separating the plane into $k < \overline{k}$ and $k > \overline{k}$, the joint effect of increasing a_i on players $-i$ is zero.

Increasing the military strength of one agent can thus act as a deterrent to conflict in small or sparse conflict networks. In large or dense conflict networks, raising one player's conflict technology increases infighting on the remaining conflicts, which are numerous and thus increases total conflict intensity and resulting damage and casualties.

1.5 Conclusion

We presented a model of conflict networks, focussing on asymmetry of parameters and changes in individual behaviour and how these affect total conflict intensity. Existence of a finite number of locally unique, interior equilibria is guaranteed. Under symmetry of parameters this extends to global uniqueness. It is possible to obtain comparative statics with respect to efficiencies and valuations. We interpret these results in terms of Bandwagoning – following a strong player against her opponents – and Balancing – where many weak(er) players join forces to oppose a strong(er) player. In the canonical case, conflict intensity become greater with asymmetry in dense networks, while asymmetry can serve as a deterrent to conflict in sparse networks.

The results regarding individual behaviour seem to advocate Bandwagoning over Balancing. Since part of these considerations also have to do with threats that are dynamic in nature, a full discussion of the two phenomena needs a model with multiple periods of interaction, although the channel we describe here should not cease to exist in any such model.

Budget constraints have an intuitive interpretation in this setting, since military budgets are likely to be fixed, at least in the short run. In a model with a strict budget constraint it is possible to characterise 'discrete comparative statics' with respect to the concrete network structures. The next chapter demonstrates a version of this model under an exogenous budget constraint.

Irrespective of the setting of costs or budgets, a natural step will be to see with whom players engage in conflict in the first place. Endogenous network formation conditional on asymmetries in technology and preferences will thus be our future focus. Since for any method akin to backward induction we require payoffs, this is technically challenging.

Providing the players with a conflict technology only, makes it hard to talk about the potential for peace in this framework. Multi-Graph theory allows for two separate networks, one with conflict and one with cooperative links. The opportunity costs of conflict generated by the opportunity of cooperation can add a new perspective to this line of research. There is a recent, special interest in this type of settings, following the work by Jackson and Nei (2015), Hiller (2017), and König et al. (2017). However, due to the complexity of applying multiple networks simultaneously, these models either focus on endogenous network formation without an explicit allocation stage, or on unidimensional action spaces over an exogenous multi-layer network.

Finally, with sufficiently simple networks, it is possible to test how real entities behave under certain parameter constellations. First steps have been made in that direction experimentally, but the specific hypotheses emerging from this paper are yet to be tested.

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1.A Proofs

For ease of notation, throughout the appendix, let us state the first order conditions and the corresponding Hessian as primitives to the proofs. For each $i \in \mathcal{I}$ we have

$$F_{ij} = \frac{\partial p(a_i x_{ij}, a_j x_{ji})}{\partial(a_i x_{ij})} a_i v_{ij} - C'(X_i) = 0 \quad \forall j \in N_i \tag{1.8}$$

The Hessian H of $F = (F_{ij}|(ij) \in B)$ is then a block-symmetric matrix for which each diagonal block associated with some player i's first order condition is given by

$$H_i = [h_i]_{lq} = \begin{cases} \frac{\partial^2 p(a_i x_{ij}, a_j x_{ji})}{(\partial a_i x_{ij})^2} a_i^2 v_{ij} - C''(X_i) & \text{if } l = q \\ -C''(X_i) & \text{else} \end{cases} \tag{1.9}$$

Each off-diagonal block in row i and column j obtains as

$$O_{ij} = [o_{ij}]_{lq} = \begin{cases} \frac{\partial^2 p(a_i x_{ij}, a_j x_{ji})}{\partial(a_i x_{ij})\partial(a_j x_{ji})} a_i a_j v_{ij} & \text{if } l = i \wedge q = j \\ 0 & \text{else} \end{cases} \tag{1.10}$$

Note that due to $p_{ij}^{12} = -p_{ji}^{21}$, we have $O_{ij}^T = -\frac{v_{ij}}{v_{ji}} O_{ji}$.

Proof of Proposition 1.1

We first give a brief summary of the proof, which proceeds in five lemmas. First, we will show that a pure strategy equilibrium exists for all $\boldsymbol{\omega} \in \Omega$. Second, by means of contradiction, we show that every such equilibrium must be strictly interior and is bounded. Third, we show that the determinant of H is strictly positive at any equilibrium. Fourth, we use a result due to Rosen (1965) according to which the equilibrium is unique if

$$\sigma(x, r) = \sum_{i \in \mathcal{I}} r_i \pi_i, \quad r_i \geq 0 \tag{1.11}$$

is 'strictly diagonally concave'. Goodman (1980) shows that a sufficient condition for this within our setting is that π_i is concave in $\boldsymbol{x}_i$ and convex in $\boldsymbol{x}_{-i}$ for all $i \in \mathcal{I}$ and that $\sigma(x, r)$ is concave in $\boldsymbol{x}$. We show that this is true at any $\boldsymbol{\omega}$ such that $v_{ij} = v_{ji}$ for all $(ij) \in B$ and thus the uniqueness result of Franke and Öztürk (2015) carries over to our setting. Due to the earlier application of the IFT at that point, it follows that there exists a neighbourhood around any symmetric parametrisation. The resulting matrix of comparative statics in expression (1.2) holds for any equilibrium. We again apply the implicit function theorem to show local uniqueness for all other equilibria.

Fifth, to show finiteness of these locally unique equilibria we demonstrate that the set of equilibria is compact. In this compact set there cannot be an (infinite) sequence of equilibria, as this would lead to a contradiction with local uniqueness.

Lemma 1.1.
A pure strategy equilibrium exists for all $\boldsymbol{\omega} \in \Omega$.

Proof. Applying the well-known theorems due to Debreu (1952), Fan (1952) and Glicksberg (1952), the result follows from making the following assertions. The game with the CSF defined in footnote 17 is a continuous game with a finite set of players.
The general formula for $\det(H_i)$ obtains as

$$\det(H_i) = \left(\prod_{j \in N_i} a_i^2 p_{ij}^{11} v_{ij}\right) - C''(X_i) \left(\sum_{j \in N_i} \prod_{l \neq j} a_i^2 p_{il}^{11} v_{il}\right). \tag{1.12}$$

To see this, consider the 4×4 case with $\mathfrak{a}_j = a_i^2 p_{ij}^{11} v_{ij}$ and $\mathfrak{b} = C''(X_i)$:

$$\det(H_i) = \det \begin{pmatrix} \mathfrak{a}_1 - \mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & \mathfrak{a}_2 - \mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & -\mathfrak{b} & \mathfrak{a}_3 - \mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} & \mathfrak{a}_4 - \mathfrak{b} \end{pmatrix}$$

Multiplying each row $j > 1$ with $\dfrac{\mathfrak{a}_1}{\mathfrak{a}_j}$ and adding the resulting rows to the first, we get

$$\begin{aligned}
\det(H_i) &= \det(H_i') \\
&= \det \begin{pmatrix} \mathfrak{a}_1 - \mathfrak{b}\sum_{j=2}^4 \frac{\mathfrak{a}_1}{\mathfrak{a}_j} & \mathfrak{a}_1 - \mathfrak{b}\sum_{j=2}^4 \frac{\mathfrak{a}_1}{\mathfrak{a}_j} & \mathfrak{a}_1 - \mathfrak{b}\sum_{j=2}^4 \frac{\mathfrak{a}_1}{\mathfrak{a}_j} & \mathfrak{a}_1 - \mathfrak{b}\sum_{j=2}^4 \frac{\mathfrak{a}_1}{\mathfrak{a}_j} \\ -\mathfrak{b} & \mathfrak{a}_2 - \mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & -\mathfrak{b} & \mathfrak{a}_3 - \mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} & \mathfrak{a}_4 - \mathfrak{b} \end{pmatrix} \\
&= \left[\mathfrak{a}_1 - \mathfrak{b}\sum_{j=2}^4 \frac{\mathfrak{a}_1}{\mathfrak{a}_j}\right] \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ -\mathfrak{b} & \mathfrak{a}_2 - \mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & -\mathfrak{b} & \mathfrak{a}_3 - \mathfrak{b} & -\mathfrak{b} \\ -\mathfrak{b} & -\mathfrak{b} & -\mathfrak{b} & \mathfrak{a}_4 - \mathfrak{b} \end{pmatrix} \\
&= \left[\mathfrak{a}_1 - \mathfrak{b}\sum_{j=1}^4 \frac{\mathfrak{a}_1}{\mathfrak{a}_j}\right] \det \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \mathfrak{a}_2 & 0 & 0 \\ 0 & 0 & \mathfrak{a}_3 & 0 \\ 0 & 0 & 0 & \mathfrak{a}_4 \end{pmatrix}
\end{aligned}$$

$$= \left[\mathfrak{a}_1 - \mathfrak{b} \sum_{j=1}^{4} \frac{\mathfrak{a}_1}{\mathfrak{a}_j} \right] \prod_{l=2}^{4} (\mathfrak{a}_l)$$
$$= \prod_{q=1}^{4} \mathfrak{a}_q - \mathfrak{b} \sum_{j=1}^{4} \prod_{l \neq j} \mathfrak{a}_l.$$

The first equality is due to determinants being invariant to row and column operations. The second equality follows from netting out the common factor in the first row. The third equality obtains from multiplying the first row with $\mathfrak{b}$ and adding it to all other rows. The resulting matrix only has one non-zero permutation for the diagonal product, which leads to the next line. Simplifying this and plugging in the terms for $\mathfrak{a}$ and $\mathfrak{b}$, results in the formula for the determinant. It should be easy to see that this generalises to any size of the matrix, providing formula (1.12).

Note that this general form also applies to any of H_i's leading principal minors of order k, denoted H_i^k from the way we derived it. Note also, that whatever the sign of $\det(H_i)$, deleting one factor $\mathfrak{a}_h$ from each product changes the sign of the expression, as $p_{ij}^{11} < 0$. Thus, since the determinant of H_i is positive whenever d_i is even and negative whenever d_i is odd and the signs of its principal minors are alternating, the claim follows as this shows that H_i is negative definite. □

Lemma 1.2.

In any equilibrium we have $x_{ij} \in [\epsilon_i^*, M_i]$ for some finite $M_i > 0$ and some small but positive ϵ_i for all $i, j\mathcal{I}$.

Proof. We need to verify three claims.

Claim 1.1.

In every equilibrium, there exists a bound M_i for all $i \in \mathcal{I}$ such that for every $j \in N_i$ we have $x_{ij} < M_i$.

Consider the highest possible revenue a player can get when winning all conflicts.

$$V_i := \sum_{j \in N_i} v_{ij}$$

Player i's effort levels are thus bounded by $M_i = C^{-1}(V_i)$, for otherwise her payoff would be negative and exerting zero efforts would result in a higher payoff.

Claim 1.2.

Any strategy profile with $x_{ij} = x_{ji} = 0$ for any $(ij) \in B$ can never be an equilibrium.

Suppose it can. Player i's marginal utility for these efforts under the CSF defined in footnote 17, is given by

$$\frac{\delta}{\delta^2} = \frac{1}{\delta}.$$

Since her effort levels are bounded above by M_i, her highest marginal costs are $C'(M_i)$. We can thus always find a δ^* such that for any $\delta < \delta^*$ we have

$$\frac{1}{\delta} > C'(M_i).$$

Thus, there is a profitable deviation. A contradiction.

Claim 1.3.
Any strategy profile with $x_{ij} > 0$ and $x_{ji} = 0$ for any $(ij) \in B$ can never be an equilibrium.

Suppose not and let player i's strategy profile be given by $x_i = (x_{i1}, ..., x_{ij}, ..., x_{in_i})$. Now consider the alternative profile x_i' which is such that $x_{ij}' = x_{ij} - \epsilon > 0$. The probability of winning on (ij) is still 1 and costs have reduced, thus it constitutes a profitable deviation. A contradiction.
We can thus find some ϵ_i for each player, such that $x_{ij} > \epsilon_i$ for all $j \in N_i$. $\square$

Lemma 1.3.
For every $\boldsymbol{\omega} \in \Omega$ we have $\det(H) > 0$.

Proof. Besides the diagonal blocks, H is a sparse matrix with only one (potentially) non-zero element in each O_{ij}. The determinant can thus be expressed as the sum of the determinant of the diagonal matrix and the additional possible permutations with the respective rows. Each of these possible permutations that leaves a non-zero diagonal product is associated with one or more conflicts.

As a minimal example, consider H associated with the line network with three players:

$$\left(\begin{array}{c|cc|c}
a_1^2 p_{12}^{11} v_{12} - C''(X_1) & a_1 a_2 p_{12}^{12} v_{12} & 0 & 0 \\
\hline
-a_1 a_2 p_{12}^{12} v_{21} & a_2^2 p_{21}^{11} v_{21} - C''(X_2) & -C''(X_2) & 0 \\
0 & -C''(X_2) & a_2^2 p_{23}^{11} v_{23} - C''(X_2) & a_2 a_3 p_{23}^{12} v_{23} \\
\hline
0 & 0 & -a_2 a_3 p_{23}^{12} v_{32} & a_3^2 p_{32}^{11} v_{32} - C''(X_3)
\end{array}\right)$$

The diagonal blocks are H_1, H_2 and H_3 respectively. We can see that the only way to swap rows without moving a zero into the diagonal is choosing rows corresponding to the same conflict. For example, rows 1 and 2 correspond to the battlefield between players 1 and 2 and rows 3 and 4 correspond to the conflict between players 2 and 3.

$$\left(\begin{array}{c|cc|c}
-a_1a_2p^{12}_{12}v_{21} & a_2^2p^{11}_{21}v_{21}-C''(X_2) & -C''(X_2) & 0 \\
\hline
a_1^2p^{11}_{12}v_{12}-C''(X_1) & a_1a_2p^{12}_{12}v_{12} & 0 & 0 \\
0 & -C''(X_2) & a_2^2p^{11}_{23}v_{23}-C''(X_2) & a_2a_3p^{12}_{23}v_{23} \\
\hline
0 & 0 & -a_2a_3p^{12}_{23}v_{32} & a_3^2p^{11}_{32}v_{32}-C''(X_3)
\end{array}\right)$$

When we do the first of these swaps, the diagonal product obtains as $-\left(p^{12}_{12}\right)^2 v_{12}v_{21}$ times the determinants of all blocks after deleting the corresponding cofactors, i.e., the first row <u>and</u> column and the second row <u>and</u> column.

$$\left(\begin{array}{c|cc|c}
\cancel{a_1^2p^{11}_{12}v_{12}-C''(X_1)} & \cancel{a_1a_2p^{12}_{12}v_{12}} & \cancel{0} & \cancel{0} \\
\hline
\cancel{-a_1a_2p^{12}_{12}v_{21}} & \cancel{a_2^2p^{11}_{21}v_{21}-C''(X_2)} & \cancel{-C''(X_2)} & \cancel{0} \\
\cancel{0} & \cancel{-C''(X_2)} & a_2^2p^{11}_{23}v_{23}-C''(X_2) & a_2a_3p^{12}_{23}v_{23} \\
\hline
\cancel{0} & \cancel{0} & -a_2a_3p^{12}_{23}v_{32} & a_3^2p^{11}_{32}v_{32}-C''(X_3)
\end{array}\right)$$

Let ϕ_{12} be the collection of all rows and columns (here 1 and 2) that have been deleted and $H_i(\phi_{12})$ is the resulting block for matrix H_i (the bottom right 4×4 matrix). We see that $H_1(\phi_{12})$ is a 0×0 matrix with determinant 1, since the top left block has been crossed out. The determinant of $H_2(\phi_{12})$ is simply $a_2^2p^{11}_{23}v_{23} - C''(X_2)$ and $H_3(\phi_{12}) = H_3$ still has determinant $a_3^2p^{11}_{32}v_{32} - C''(X_3)$. Thus, for ϕ_{12} the product of the diagonal element obtains as

$$-a_1a_2\left(p^{12}_{12}\right)^2 v_{12}v_{21}\det(H_1(\phi_{12}))\det(H_2(\phi_{12}))\det(H_3(\phi_{12})).$$

Since the number of permutations encoded in ϕ_{12} (which is one, as we only swapped rows once) is odd, this product gets multiplied by (-1) when added to the sum that constitutes the determinant. To get the full determinant we would have to also consider ϕ_{23} and $\{\phi_{12}, \phi_{23}\}$ and the diagonal product of all determinants of the individual blocks.

This logic applies more generally. Let the set of all permutations and their combinations be denoted S_n with typical element ϕ. It contains all sets of additional row permutations that correspond to some set of conflicts $(ij) \in B$.[31] The signum function sgn(ϕ) is negative when $|\phi|$ is odd and positive when $|\phi|$ is even, where $|\phi|$ is the number of permutations in ϕ.

[31] When rows i and j are swapped, this is counted as one permutation.

Using the Leibniz formula for determinants, we get

$$\begin{aligned}
\det(H) =& \prod_{i\in\mathcal{I}} \det(H_i) + \sum_{\phi\in S_n} \text{sgn}(\phi) \prod_{(ij)\in\phi} -a_i a_j \left(p_{ij}^{12}\right)^2 v_{ij}v_{ji} \prod_{i\in\mathcal{I}} \det(H_i(\phi)) \\
=& \prod_{i\in\mathcal{I}} \det(H_i) + \sum_{\phi\in S_n} \text{sgn}(\phi)(-1)^{|\phi|} \prod_{(ij)\in\phi} a_i a_j \left(p_{ij}^{12}\right)^2 v_{ij}v_{ji} \prod_{i\in\mathcal{I}} \det(H_i(\phi)) \\
=& \prod_{i\in\mathcal{I}} \det(H_i) + \sum_{\phi\in S_n} \prod_{(ij)\in\phi} a_i a_j \left(p_{ij}^{12}\right)^2 v_{ij}v_{ji} \prod_{i\in\mathcal{I}} \det(H_i(\phi)).
\end{aligned}$$

We know that $\prod_{i\in\mathcal{I}} \det(H_i) > 0$ as all H_i associated with an even number of conflicts have $\det(H_i) > 0$ and all H_i associated with an odd number of conflicts have $\det(H_i) < 0$, due to negative definiteness. But since the total number of effort choices is $2b$, there must be an even number of the latter type of H_i, thus $\prod_{i\in\mathcal{I}} \det(H_i) > 0$ is true.[32] As argued earlier, whenever we delete a row and column j of any H_i, the sign changes. But since the considered permutations always affect exactly two such Hessians, the sign of $\prod_{i\in\mathcal{I}} \det(H_i(\phi))$ for any combination of permutations in ϕ cannot change. Since $\prod_{(ij)\in\phi} a_i a_j \left(p_{ij}^{12}\right)^2 v_{ij}v_{ji} \geq 0$, this shows that $\det(H) > 0$ for all $\omega \in \Omega$. □

Lemma 1.4.
Every equilibrium is locally unique and the comparative statics at any equilibrium are given by expression (1.2)*. Furthermore, for an open neighbourhood around any symmetric parametrisation, there exists a globally unique, interior, pure-strategy Nash Equilibrium.*

Proof. The first sentence follows immediately from applying the Implicit Function Theorem (IFT). Since $\det(H) > 0$ and thus $\det(H) \neq 0$, and F being continuously differentiable on $\mathbb{R}^{4b+n}$, it implies that the solution at any equilibrium is locally continuous in its parameters and that the derivatives are given by expression (1.2).

For the second part, we apply the result from Rosen (1965) and Goodman (1980) at an arbitrary symmetric equilibrium. The individual payoff functions are strictly concave in own strategies (H_i is negative definite for all $\boldsymbol{x} \in \mathbb{R}_+^{2b}$) and strictly convex in the strategies of others, as each payoff function for player i is the sum of convex CSFs in $\boldsymbol{x}_{-i}$. Function (1.11)

[32] Also see the handshaking lemma, which is a well-known and intuitive result in network economics.

for $r_i = r$ for all $i \in \mathcal{I}$ can be rewritten as

$$\begin{aligned}\sigma(\boldsymbol{x}, \boldsymbol{r}) &= \sum_{i\in\mathcal{I}} r p_{ij} v_{ij} - \sum_{i\in\mathcal{I}} rC(X_i) = r\sum_{i\in\mathcal{I}} p_{ij} v_{ij} - r\sum_{i\in\mathcal{I}} C(X_i)\\ &= r\sum_{(ij)\in B} (p_{ij}v_{ij} + (1-p_{ij})v_{ji}) - r\sum_{i\in\mathcal{I}} C(X_i)\\ &= r\sum_{(ij)\in B} v_{ji} + r\sum_{(ij)\in B} p_{ij}(v_{ij} - v_{ji}) - r\sum_{i\in\mathcal{I}} C(X_i)\\ &= r\sum_{(ij)\in B} v_{ji} - r\sum_{i\in\mathcal{I}} C(X_i),\end{aligned}$$

where the last equality follows from symmetry of conflict valuations ($v_{ij} = v_{ji}$ for all $(ij) \in B$). Since the cost functions are strictly concave for every $\boldsymbol{x} \in \mathbb{R}^{2b}_+$ and the first term is a constant, this function is strictly concave. Thus, any symmetric parameterisation has a globally unique equilibrium. Applying the IFT to any such equilibrium, using the above arguments about F and the determinant of its Jacobian H, it follows that there exists a neighbourhood of parameters for which a globally unique equilibrium exists. □

Lemma 1.5.
The number of equilibria is finite.

Proof. Finally, we want to argue that the number of equilibria is finite. First, remember that efforts are bounded. Since this is true for every effort in every equilibrium, the set of equilibria is a compact set (closed and bounded in $\mathbb{R}^{2b}$). Now suppose the number of equilibria is infinite. We could thus construct a (non-trivial) sequence of equilibria in this set. Due to the boundedness of that sequence, there exists a convergent subsequence. Since the set is compact, and thus closed, the limit of this sequence lies in the set. The contradiction occurs from noting that convergence in a metric space is Cauchy, i.e. $\forall \epsilon > 0, \exists\, N(\epsilon)$ s.th. $\forall m, n > N(\epsilon), d(x_m, x_n) < \epsilon$, and that the limit point is locally unique as it is part of the set for which $\det(H) > 0$. We would have found a locally unique equilibrium around which every open ϵ-ball contains an infinite number of distinct equilibria (with that same property). A contradiction. Thus, there cannot exist an infinite number of equilibria. □

■

Proof of Proposition 1.2

As the network of conflict $\mathcal{G}$ induced by the disjoint pair of sets $(\mathcal{I}, B)$ is a connected graph we always can find a path $\mathcal{P}$ between any two nodes. Let us take any two nodes h and l that are not rivals (i.e. $h \notin N_l$ and $l \notin N_h$). We know that there exist a path $\mathcal{P}_{hl} =$

$\{hi_2, i_2i_3, \ldots, i_{k-1}i_k, i_kl\}$ where $i_j \in \mathcal{I}$ and all $(i_ji_k) \in B$ between them. As usual, the best-response $\mathbf{x}_h^*$ of player h depends on her rivals' actions, thus $\mathbf{x}_h^* = \mathbf{x}_h(\mathbf{x}_{r_1}^*, \ldots, \mathbf{x}_{r_j}^*, \ldots, \mathbf{x}_{r_k}^*)$, where all $r_j \in N_h$. In particular, we know that $(hi_2) \in B$ which implies that $i_2 \in N_h$, them $\mathbf{x}_h^* = \mathbf{x}_h(\mathbf{x}_{r_1}^*, \ldots, \mathbf{x}_{i_2}^*, \ldots, \mathbf{x}_{r_j}^*, \ldots, \mathbf{x}_{r_k}^*)$. Notice that $\mathbf{x}_{i2}^* = \mathbf{x}_{i_2}(\mathbf{x}_{g_1}^*, \ldots, \mathbf{x}_{i_3}^*, \ldots, \mathbf{x}_{g_j}^*, \ldots, \mathbf{x}_{g_k}^*)$ for all $g_i \in N_{i_2}$, since i_3 is an opponent of i_2. Following the sequence of nodes describe path the path $\mathcal{P}_{hl}$, we can rewrite the best-response of player h as a function of her direct rivals and all the nodes in the path such that $\mathbf{x}_h^* = \mathbf{x}_h(\mathbf{x}_{r_1}^*, \ldots, \mathbf{x}_{r_j}^*, \ldots, \mathbf{x}_{r_k}^*, \mathbf{x}_{i2}^*, \mathbf{x}_{i3}^*, \ldots, \mathbf{x}_{ik}^*, \mathbf{x}_l^*)$. Thus, even though players are not direct rivals and they are pay-off irrelevant to each other in the primitives of the game, the equilibrium effort choice of any player depends on the action of any player that is connected with her through a path. As the graph is connected, this it true for any pair of players. Therefore, we have $\mathbf{x}_h^* = \mathbf{x}_h(\mathbf{X}_{-\{h,l\}}^*, \mathbf{x}_l^*)$ and analogously $\mathbf{x}_l^* = \mathbf{x}_l(\mathbf{X}_{-\{h,l\}}^*, \mathbf{x}_h^*)$ where $\mathbf{X}_{-\{h,l\}}^* = [\mathbf{x}_i^*]_{i \in \mathcal{I} \setminus \{h,l\}}$. ■

Proof of Corollary 1.1

Recall the first order conditions induced by the optimisation problem of player i in conflict (ij),

$$\begin{aligned}
&\frac{a_i v_{ij} f'(a_i x_{ij}) f(a_j x_{ji})}{[f(a_i x_{ij}) + f(a_j x_{ji})]^2} = C'(X_i) \\
\Rightarrow & f(a_i x_{ij})^2 + 2f(a_i x_{ij}) f(a_j x_{ji}) + f(a_j x_{ji})^2 - \frac{a_i v_{ij} f'(a_i x_{ij}) f(a_j x_{ji})}{C'(X_i)} = 0
\end{aligned}$$

Based on the result presented in Proposition 1.2, we know that player i's allocation in the conflict against j depends also on the actions of j's rivals. If $k \in N_j$, player j's optimality condition requires that

$$f(a_i x_{ij}) = \frac{[f(a_i x_{ij}) + f(a_j x_{ji})]^2}{[f(a_k x_{kj}) + f(a_j x_{jk})]^2} \frac{f(a_k x_{kj}) f'(a_j x_{jk})}{f'(a_j x_{ji})} \frac{v_{jk}}{v_{ji}}$$

Using this expression in the optimality condition of player i for conflict (ij), we get

$$\begin{aligned}
&\left[\frac{[f(a_i x_{ij}) + f(a_j x_{ji})]^2}{[f(a_k x_{kj}) + f(a_j x_{jk})]^2} \frac{f(a_k x_{kj}) f'(a_j x_{jk})}{f'(a_j x_{ji})} \frac{v_{jk}}{v_{ji}}\right]^2 \\
&\qquad + 2\left[\frac{[f(a_i x_{ij}) + f(a_j x_{ji})]^2}{[f(a_k x_{kj}) + f(a_j x_{jk})]^2} \frac{f(a_k x_{kj}) f'(a_j x_{jk})}{f'(a_j x_{ji})} \frac{v_{jk}}{v_{ji}}\right] f(a_j x_{ji}) + \\
&\qquad\qquad f(a_j x_{ji})^2 - \frac{a_i v_{ij} f'(a_i x_{ij}) f(a_j x_{ji})}{C'(X_i)} = 0
\end{aligned}$$

We are interested to solve for $f(a_i x_{ij})$ and $f(a_j x_{ji})$. To do that we need to find the roots of the above polynomial. Notice that this expression is of the form

$$A\ f(a_i x_{ij})^4 + B\ f(a_i x_{ij})^3 f(a_j x_{ji}) + C\ f(a_i x_{ij})^2 (2f(a_j x_{ji}) + 6f(a_j x_{ji})^2) + \\ D\ f(a_i x_{ij})(4f(a_j x_{ji})^3 + 4f(a_j x_{ji})^2) + E\ 2f(a_j x_{ji})^3 + F\ f(a_j x_{ji})^4 + C = 0,$$

which independently of the forms of the scalar A, B, C, D, E and F is irreducible over $\mathbb{C}$ and therefore irreducible over $\mathbb{R}_+$. We can continue the substitution process along any path between player i and any other player $l \in \mathcal{I}$. Each step further we are going to find a new term in our polynomial. The exponent of this term is going to be squared due to the non-linearity of the primitives of the contest success function. Thus, to solve the system of equations induced by the maximisation problem of each individual, we need to solve at least one polynomial of degree 2^L where L is the largest path between a pair of player i and j. Hence, to solve the system of equations we need to find the root of at least one polynomial of the form

$$ax^{2^L} + bx^{2^L - 1} + \ldots + cx^{2^2 - 1} + dx^{2^1 - 1} + e = 0$$

Therefore, network structures in which we can find a path of length greater than or equal to 3 require to find the roots of at least one general algebraic equation of degree higher or equal to 8 in the best case scenario.

Theorem 1.1.
Abel-Ruffini Theorem (1779)
A general algebraic equation of degree ≥ 5 *cannot be solved in radicals. This means that there does not exist any formula which would express the roots of such equation as functions of the coefficients by means of the algebraic operations and roots of natural degrees.*

By looking at the functional form of the reaction functions and the result of the Abel-Ruffini Theorem, we can say that this type of system does not have an algebraic solution using radicals. Hence, in our setting the equilibrium of the game does not have a generic, algebraic solution if the longest path between any two players is higher than 3. ■

Proof of Proposition 1.3

$\Rightarrow$: Let $\boldsymbol{\omega} = \{\mathbf{1}_{2b}\overline{v}, \mathbf{1}_n\overline{a}\}$ for some $a, v \in \mathbb{R}_{++}$ and $d_i = k \in \mathbb{N}_+$ for all $i \in \mathcal{I}$.
Consider some FOC of the maximisation problem for some player i and some conflict (ij)

$$\frac{\partial p(\overline{a}x_{ij}, \overline{a}x_{ji})}{\partial(\overline{a}_i x_{ij})}\overline{a}v = C'(X_i)$$

Assuming symmetry $x_{ij} = x_{ji} = x^s$ gives

$$x^s = \frac{1}{k} C'^{-1}(p^1(\overline{a}x^s, \overline{a}x^s)av)$$

It is unique as per proposition 1.1.

$\Leftarrow$: Assume $x_{lq} = x^s$ for all $(lq) \in B$ some x^s. Redoing the steps in the first part for any two players i and j, we get

$$\frac{1}{d_i} C'^{-1}(p^1(\overline{a}x^s, \overline{a}x^s)\overline{a}v) = \frac{1}{d_j} C'^{-1}(p^1(\overline{a}x^s, \overline{a}x^s)\overline{a}v),$$

which implies $d_i = d_j$. ∎

Proof of Proposition 1.4

In this case the Jacobian of the system of FOCs contains the following elements:

$$\begin{aligned}
\frac{\partial F_{ij}}{\partial x_{ij}} &= a_i^2 p_{ij}^{11} v_{ij} - C''(X_i) < 0 \\
\frac{\partial F_{ij}}{\partial x_{ji}} &= a_i a_j p_{ij}^{12} v_{ij} \lesseqgtr 0 \\
\frac{\partial F_{ij}}{\partial x_{iq}} &= -C''(X_i) < 0 \\
\frac{\partial F_{ij}}{\partial x_{qi}} &= 0
\end{aligned}$$

Since at any strictly symmetric equilibrium in a k-regular network $p_{ij}^{12} = 0$ for all $(ij) \in B$, this results in $D_x(F) = \text{diag}(A_1, A_2, ..., A_n)$ with $A_i = B_i + E_i$ and

$$B_i = \begin{pmatrix} z_{i1} & 0 & \cdots & \cdots & 0 \\ 0 & z_{i2} & 0 & \cdots & 0 \\ \vdots & & \ddots & & \\ \vdots & & & \ddots & \\ 0 & & & & z_{iN} \end{pmatrix}$$

with $z_{ij} := a_i^2 p_{ij}^{11} v_{ij}$ and $E_i = [e]_{ql} = -C''(X_i)$ for all (ql). Note that at the strictly symmetric equilibrium in a k-regular network we have $z_{ij} = z_{ql} = z = \overline{a}^2 p_{ij}^{11} \overline{v}$.
The inverse of this matrix is given by applying the Sherman-Morrison formula.

$$A_i^{-1} = \frac{1}{z} I - \frac{\frac{1}{z^2} E}{1 - \frac{1}{z} d_i C''(X)}$$

In a more compact way,

$$A^{-1} = G = [g]_{l,q} = \begin{cases} \frac{z-(k-1)C''(X)}{z-kC''(X)} z^{-1} & \text{if } l = q \\ \frac{C''(X)}{z-kC''(X)} z^{-1} & \text{else.} \end{cases}$$

The partial effects are then given by

$$\frac{\partial x}{\partial \boldsymbol{\omega}} = -\left[D_x(F)\right]^{-1} D_{\boldsymbol{\omega}}(F)$$

or more precisely:

$$\begin{aligned} \frac{\partial x_{ij}}{\partial v_{ij}} &= -\frac{z-(k-1)C''(X)}{z-kC''(X)} \frac{p^1}{\overline{a}p^{11}\overline{v}} > 0 \\ \frac{\partial x_{iq}}{\partial v_{ij}} &= -\frac{C''(X)}{z-kC''(X)} \frac{p^1}{\overline{a}p^{11}\overline{v}} < 0 && \text{for } q \neq j \\ \frac{\partial x_{ij}}{\partial a_i} &= -\frac{1+z}{z-kC''(X)} \left(\frac{p^1}{\overline{a}^2 p^{11}} + \frac{\overline{x}}{\overline{a}} \right) \end{aligned}$$

■

Proof of Proposition 1.5

Recall the expression in parenthesis in (1.4).

$$\frac{p^1}{p^2} \frac{\overline{v}}{\overline{a}^2 \overline{v}} + \frac{x^s}{\overline{a}} \tag{1.13}$$

Given the assumed functional forms, we have

$$p^1(x^s) = \frac{r}{4} \frac{1}{\overline{a}x^s}$$

and

$$p^2(x^s) = -\frac{r}{4} \frac{1}{(\overline{a}x^s)^2}.$$

Thus, we have

$$\frac{p^1}{p^2} = -\overline{a}x^s,$$

which implies

$$\frac{p^1}{p^2} \frac{\overline{v}}{\overline{a}^2 \overline{v}} + \frac{x^s}{\overline{a}} = 0, \tag{1.14}$$

which is the claim of the proposition. ■

Proof of Proposition 1.6

From the derivatives obtained in the earlier result, we know already that $x_{ij}(\boldsymbol{\omega}') > x^s$ as well as $x_{ik}(\boldsymbol{\omega}') < x^s$ for all $k \neq j$. Note that the change induced in a nested function f is less than that induced in g, whenever

$$\begin{aligned}|Df(g(x))g'(x)| &< |g'(x)|\\ |Df(g(x))||g'(x)| &< |g'(x)|\\ |Df(g(x))| &< 1.\end{aligned}$$

The changes we consider are either $\frac{\partial x_{ij}}{\partial x_{ji}}$, which is zero at the symmetric parametrisation and close to zero near it, and

$$\left|\frac{\partial x_{ij}}{\partial x_{ik}}\right| = \left|-\frac{-C''(X_i)}{a_1^2 p^{11} v_{ij} - C''(X_i)}\right| = \left|\frac{C''(X_i)}{C''(X_i) - a_1^2 p^{11} v_{ij}}\right| < 1.$$

This implies that any effect of a sufficiently small change in parameters from $\overline{\boldsymbol{\omega}}$ diminishes over with increasing length of a path.
We denote each best response function as a nested function of the strategies that constitute the shortest path through the graph to a nonzero derivative. In a slight abuse of notation, let us denote player i's best response function on conflict (ij) as $x_{ij}(x_{ji}(a_i))$. Using a second-order Taylor approximation, we get

$$\begin{aligned}x_{ji}(x_{ij}(a_i)) =& x_{ji}(x_{ij}(\overline{a})) + \frac{\partial x_{ji}}{\partial x_{ij}}\frac{\partial x_{ij}}{\partial a_i}(\overline{a})(a_i - \overline{a})\\ &+ \frac{1}{2}\left(\frac{\partial^2 x_{ji}}{(\partial x_{ij})^2}\left(\frac{\partial x_{ij}}{\partial a_i}\right)^2 + \frac{\partial x_{ji}}{\partial x_{ij}}\frac{\partial^2 x_{ij}}{(\partial a_i)^2}\right)(\overline{a})(a_i - \overline{a})^2\\ |_{\frac{\partial x_{ji}}{\partial x_{ij}}(\overline{a})=0} =& x^s - \frac{1}{2}\frac{\overline{a}^3 p^{122}\overline{v}}{\overline{a}^2 p^{11}\overline{v} - C''(X_i)}\left(\frac{\partial x_{ij}}{\partial a_i}\right)^2 (a_i - \overline{a})^2 < x^s.\end{aligned}$$

Note that this is irrespective of the sign of $\frac{\partial x_{ij}}{\partial a_i}$. Similarly, it follows that

$$x_{jk}(x_{ji}(x_{ij}(a_i))) = x^s + \frac{\partial x_{jk}}{\partial x_{ji}}\frac{\partial^2 x_{ji}}{(\partial x_{ij})^2}\left(\frac{\partial x_{ij}}{\partial a_i}\right)^2 (a_i - \overline{a})^2 > x^s.$$

■

Proof for Proposition 1.7

Just as in the above proof the effects of the other players in S can be obtained via a Taylor approximation as

$$x_{ji}(x_{ij}(v_{ij})) \tilde{=} x^s + \frac{1}{2}\frac{\partial^2 x_{ji}}{(\partial x_{ij})^2}\left(\frac{\partial x_{ij}}{\partial v_{ij}}\right)^2 (v_{ij} - \overline{v})^2 < x^s$$

$$x_{jk}(x_{ji}(x_{ij}(v_{ij}))) \tilde{=} x^s + \frac{1}{2}\frac{\partial x_{jk}}{\partial x_{ji}}\frac{\partial^2 x_{ji}}{(\partial x_{ij})^2}\left(\frac{\partial x_{ij}}{\partial v_{ij}}\right)^2 (v_{ij} - \overline{v})^2 > x^s$$

$$x_{kj}(x_{ki}(x_{ik}(v_{ij}))) \tilde{=} x^s + \frac{1}{2}\frac{\partial x_{kj}}{\partial x_{ki}}\frac{\partial^2 x_{ki}}{(\partial x_{ik})^2}\left(\frac{\partial x_{ik}}{\partial v_{ij}}\right)^2 (v_{ij} - \overline{v})^2 > x^s$$

$$x_{ki}(x_{ik}(v_{ij})) \tilde{=} x^s + \frac{1}{2}\frac{\partial^2 x_{ki}}{(\partial x_{ik})^2}\left(\frac{\partial x_{ik}}{\partial v_{ij}}\right)^2 (v_{ij} - \overline{v})^2 < x^s$$

$$x_{kl}(x_{ik}(v_{ij})) \tilde{=} x^s + \frac{1}{2}\frac{\partial x_{kl}}{\partial x_{ki}}\frac{\partial^2 x_{ki}}{(\partial x_{ik})^2}\left(\frac{\partial x_{ik}}{\partial v_{ij}}\right)^2 (v_{ij} - \overline{v})^2 > x^s$$

Since the $|\frac{\partial x_{ij}}{v_{ij}}|, |\frac{\partial x_{ik}}{v_{ij}}| > \frac{\partial x_{lq}}{v_{ij}} = 0$ for any (lq) such that $l \neq i$, there exists some $\epsilon = v_{ij} - \overline{v}$ such that for any $A \neq 0$ any $|A\epsilon^2|$ is strictly between the absolute value of these partial derivatives and 0. ∎

Proof of Proposition 1.8

First, note that at $\overline{\omega}$ we have

$$\begin{aligned} \frac{\partial^2 x_{ij}}{(\partial x_{ji})^2} &= \frac{\partial^2 x_{lq}}{(\partial x_{ql})^2} \quad \forall l, q \in \mathcal{I} \\ \frac{\partial x_{ij}}{\partial x_{ih}} &= \frac{\partial x_{ql}}{\partial x_{qr}} \quad \forall h, l, q, r \in \mathcal{I} \end{aligned} \tag{1.15}$$

From Proposition 1.6 we know that for all $m \neq i$ with $m \in S$

$$\Delta x_{mi} = x_{mi}(a_i) - x_{mi}(\overline{a}) = \frac{1}{2}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\left(\frac{\partial x_{im}}{\partial a_i}\right)^2 (a_i - \overline{a})^2$$

Using the equalities in 1.15, this change occurs k times. Similarly, all k players $m \neq i$, $m \in S$ change their behaviour towards their $k-1$ opponents $q \neq i, m$, $q \in S$

$$\Delta x_{mq} = x_{mq}(a_i) - x_{mq}(\overline{a}) = \frac{1}{2}\frac{\partial x_{mq}}{\partial x_{mi}}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\left(\frac{\partial x_{im}}{\partial a_i}\right)^2 (a_i - \overline{a})^2$$

Finally, player i's change towards her k opponents $m \neq i$ in S is

$$\Delta x_{im} = \frac{\partial x_{im}}{\partial a_i}(a_i - \overline{a})$$

Thus, denoting $\Delta a_i = a_i - \overline{a}$, we have

$$\begin{aligned} \Delta X^s =& k\frac{\partial x_{im}}{\partial a_i}\Delta a_i + \frac{k}{2}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\left(\frac{\partial x_{im}}{\partial a_i}\right)^2 \Delta a_i^2 \\ &+ \frac{k(k-1)}{2}\frac{\partial x_{mq}}{\partial x_{mi}}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\left(\frac{\partial x_{im}}{\partial a_i}\right)^2 \Delta a_i^2 \end{aligned}$$

which ultimately gives us

$$\begin{aligned} \frac{\Delta X^s}{\Delta a_i} =& k\frac{\partial x_{im}}{\partial a_i} + \frac{k}{2}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\left(\frac{\partial x_{im}}{\partial a_i}\right)^2 \Delta a_i \\ &+ \frac{k(k-1)}{2}\frac{\partial x_{mq}}{\partial x_{mi}}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\left(\frac{\partial x_{im}}{\partial a_i}\right)^2 \Delta a_i \end{aligned}$$

If $\frac{\partial x_{ij}}{\partial a_i} > 0$ we have

$$\frac{\Delta X^s}{\Delta a_i} > 0 \Leftrightarrow 1 + \frac{1}{2}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\frac{\partial x_{im}}{\partial a_i}\Delta a_i\left(1 + (k-1)\frac{\partial x_{mq}}{\partial x_{mi}}\right) > 0$$

Since $\frac{\partial^2 x_{mi}}{(\partial x_{im})^2} < 0$, negativity of the term in parentheses guarantees that the above inequality holds. That is

$$\begin{aligned} & 1 + (k-1)\frac{\partial x_{mq}}{\partial x_{mi}} < 0 \\ \Leftrightarrow & 1 + (k-1)\frac{C''(X^s)}{p^{11} - C(X^s)} \quad |p^{11} - C''(X^s) < 0 \\ \Leftrightarrow & p^{11} + (k-2)C''(X^s) > 0 \\ \Leftrightarrow & k > 2 - \frac{p^{11}}{C''(X^s)} \end{aligned}$$

If $\frac{\partial x_{ij}}{\partial a_i} < 0$ we still have

$$\frac{\Delta X^s}{\Delta a_i} > 0 \Leftrightarrow 1 + \frac{1}{2}\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\frac{\partial x_{im}}{\partial a_i}\Delta a_i\left(1 + (k-1)\frac{\partial x_{mq}}{\partial x_{mi}}\right) < 0$$

In this case $\frac{\partial^2 x_{mi}}{(\partial x_{im})^2}\frac{\partial x_{im}}{\partial a_i} > 0$, thus for the inequality to be true, the term in parentheses must be positive, leading to $k < 2 - \frac{p^{11}}{C''(X^s)}$.

This is independent of the magnitude of Δa_i. ■

Proof of Proposition 1.9

We see that the total effect on the efforts of all players $m \in S$ such that $m \neq i, j$ against each other is negative as (for $q \neq i, j, m$)

$$\sum_{m \neq i,j} \sum_{l \neq m,i,j} \Delta x_{ml} = \frac{(k-1)(k-2)}{2} \frac{\partial x_{mq}}{\partial x_{mi}} \frac{\partial^2 x_{mi}}{(\partial x_{im})^2} \left(\frac{\partial x_{im}}{\partial v_{ij}} \right)^2 \Delta v_{ij}^2 > 0 \tag{1.16}$$

The total effect of an increase in v_{ij} on total effort is thus given by

$$\begin{aligned} &\left(\frac{\partial x_{ij}}{\partial v_{ij}} + (k-1) \frac{\partial x_{im}}{\partial v_{ij}} \right) \Delta v_{ij} \\ &+ \frac{1}{2} \frac{\partial^2 x_{ji}}{(\partial x_{ij})^2} \left(\frac{\partial x_{ij}}{\partial v_{ij}} \right)^2 \Delta v_{ij}^2 + \frac{(k-1)}{2} \frac{\partial^2 x_{ji}}{(\partial x_{ij})^2} \left(\frac{\partial x_{ij}}{\partial v_{ij}} \right)^2 \frac{\partial x_{jm}}{\partial x_{ji}} \Delta v_{ij}^2 \\ &+ \frac{k-1}{2} \frac{\partial^2 x_{ji}}{(\partial x_{ij})^2} \left(\frac{\partial x_{im}}{\partial v_{ij}} \right)^2 \Delta v_{ij}^2 + \frac{(k-1)^2}{2} \frac{\partial x_{mq}}{\partial x_{mi}} \frac{\partial^2 x_{mi}}{(\partial x_{im})^2} \left(\frac{\partial x_{im}}{\partial v_{ij}} \right)^2 \Delta v_{ij}^2 \end{aligned} \tag{1.17}$$

We see that the third row is $k-1$ times the second row. The second row is positive whenever

$$1 + (k-1) \frac{\partial x_{jm}}{\partial x_{ji}} < 0$$

which is true whenever $T(k) > 0$. ■

1.B Data and Graphs

The data we used for figure 1.3 stems from the Uppsala Conflict Data Program (UCDP). The dataset is *[a] dyad-year version of the UCDP/PRIO Armed Conflict Dataset. A dyad consists of two opposing actors in an armed conflict where at least one party is the government of a state.* (UCDP, 2018). For a more detailed description of the dataset also see Harbom et al. (2018) or Pettersson and Eck (2018).

Extrasystemic armed conflict is defined as a conflict between a state and a non-state group outside its own territory. Interstate armed conflicts are between two or more states. Internal armed conflicts are between the government of a state and one or more internal opposition groups, without intervention from other states. For Internationalised Internal conflict, intervention from other states on one or both sides is added to the definition.

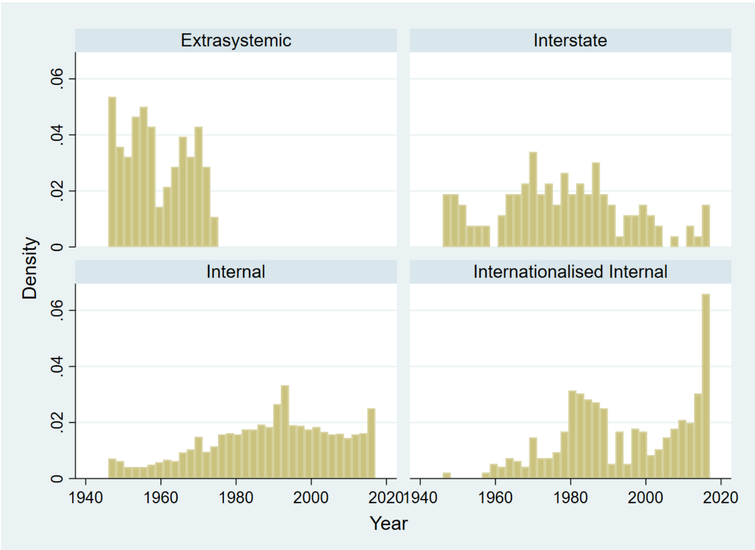

Figure 1.B.1 : Frequency of Conflict Types over Time

Chapter 2

Budget Constraints in Conflict Networks[1,2]

[1]This paper used to be a part of *Generalising Conflict Networks* (Cortes-Corrales and Gorny, 2018).

[2]We would like to thank participants from the CBESS Contest Conference 2016 and 2018 held at the University of East Anglia in Norwich and the Conflict Workshop 2018 at the University of Bath for useful comments. Also, we want to thank Sergio Currarini, David Hugh-Jones, David Rojo Arjona and Mich Tvede for useful comments. All remaining errors are our own.

Sebastián Cortes-Corrales[‡] and Paul M. Gorny[§]

Abstract

We introduce strict budget constraints into a model of conflict networks. We provide a sufficient condition for the existence of a finite number of locally unique, interior, pure strategy Nash equilibria and characterise the best response functions. Using an algorithm that seems to converge under the above sufficient condition, we investigate the robustness of earlier results and provide two examples that could not be studied in this class of models before. Heterogenous teams can outperform homogenous teams and the number of links can harm a player. we show that centrality can also hurt more distant players, since the 'enemy of my enemy's enemy is also my enemy'.

Keywords: Contest, conflict, networks, budget constraints

JEL: C72; D74; D85

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2.1 Introduction

In conflict of any type, resources employed to win the conflict are limited. Countries have military budgets agreed by the government or parliaments. Paramilitary groups are subject to resource limitations depending on how much support they get from political supporters and even in interpersonal conflict, the amount of effort is limited by physical traits. These budgets are thus central in the determination of conflict outcomes.

In hardly any conflict, there are only two parties involved. Whenever there are more than two parties, the complex constellation of their bilateral conflicts can give rise to a conflict network (Cortes-Corrales and Gorny, 2018; Franke and Öztürk, 2015). As shown previously, asymmetries can have complex effects that spread through the network. To determine their sign and magnitude is technically demanding. Introducing a budget constraint can thus help to put more structure onto the model, while allowing for (and in fact strengthening) opportunity costs between conflicts.

In settings where resources are divided into productive and defensive/appropriative resources (see e.g. Skaperdas and Syropoulos (2001) and Neary (1997)) the exact size of the split is crucial to the success of the parties' enterprise to maximise their payoffs. To allow for any such analysis with conflict networks, the second stage strategies and payoffs need to be determined, which is the aim of this chapter.

We show that the result of existence and local uniqueness of interior strategies around any symmetric parametrisation, established in the previous chapter, can be qualified with a sufficient algebraic condition.

In a canonical case, we show that best response functions can be characterised in closed form. We provide an algorithm that seems to converge to the equilibrium under the above-mentioned condition. MatLab code is provided that can calculate the equilibrium for any network structure and choice of parameters. Using it, we discuss earlier results on conflict networks and provide insights into some further interesting examples that cannot be studied as easily in the original framework.

Related Literature: We will relate to the literature on conflict networks. In the broader area our study relates to games on networks that received a lot of attention recently, both theoretically (Goeree et al., 2009; Galeotti et al., 2010; Goyal, 2012) and experimentally (Charness and Jackson, 2007; Charness et al., 2014; Kittel and Luhan, 2013). More specifically, it relates to the study of conflict games and contests on networks (Franke and Öztürk, 2015;

Herbst et al., 2015; König et al., 2015; Huremovic, 2016; Goyal et al., 2016; Cortes-Corrales and Gorny, 2018). The studies closest to this are Cortes-Corrales and Gorny (2018) and Franke and Öztürk (2015) as we are essentially taking their model and imposing a budget constraint instead of a convex cost structure.

We also relate to the study on Colonel Blotto games and crowdsourcing literature, where the use of strict budget constraints is more common. The best response functions are similar to Feldman et al. (2005), Friedman (1958) and Kovenock and Rojo-Arjona (2019). Interestingly, Feldman et al. (2005) has a model where conflicts are not necessarily bilateral and players can stay away from a conflict, while Friedman (1958) and Kovenock and Rojo-Arjona (2019) discuss a Colonel Blotto game with bilateral conflicts but only two players. The similarity of the best responses is striking and we refer to the subtleties that distinguish the models.

The rest of the chapter is structured as follows. Section 2.2 introduces the model. The sufficient condition for existence and uniqueness of interior, pure strategy equilibria is given in Section 2.3. Section 2.4 provides the best response functions and an algorithm that can solve the model. In Sections 2.5 and 2.6 we discuss the robustness of some earlier results and provide some further interesting examples. Section 2.7 concludes.

2.2 The Model

The notation is largely identical to the previous chapter. We still introduce it here again for completeness. The reader might want to skip to the bottom of page 66, where the new elements are introduced.

Let $\mathcal{I} = \{1, \ldots, n\}$ be a finite set of players with $n \geq 2$. All conflicts are contained in $B \subseteq \mathcal{I}^2$ where $\mathcal{I}^2$ is the set of unordered pairs of $\mathcal{I}$ with typical element (ij). The underlying conflict network $\mathcal{G}$ is represented by the connected graph associated with the pair of sets $(\mathcal{I}, B)$.[3] We say that any pair of players i and j is involved in a bilateral conflict (ij) if and only if $(ij) \in B$. Again, a conflict does not disappear if it is ignored and no player can be enemy of herself. Thus, we assume the network $\mathcal{G}$ to be *undirected* ($\forall i, j \in \mathcal{I} : (ij) \in B \Leftrightarrow (ji) \in B$) and *irreflexive* ($\forall i \in \mathcal{I} : (ii) \notin B$). Let $N_i = \{j \in \mathcal{I} | (ij) \in B\}$ denote the set of i's rivals. The total number of i's rivals is given by $d_i = |N_i|$. The total number of conflicts is $b = \frac{1}{2}\sum_i d_i$. We denote $S \subseteq \mathcal{I}$ a clique of $\mathcal{G}$, if every pair of players i and j in S has a conflict between them – i.e. $(ij) \in B$. In each bilateral conflict, $(ij) \in B$, players i and j fight for a

[3]A graph $\mathcal{G}$ is connected if for every pair of players i and j in $\mathcal{I}$ we can find a sequence of adjacent conflicts to travel from i to j. The results that we are presenting hold for any non-trivial component of any disconnected network. Therefore, we do not consider such network structures in our analysis.

strictly positive exogenous prize. Player i's valuation of winning the prize against player j is denoted $v_{ij} > 0$. As in Chapter 1, we allow valuations to differ across conflicts and players (even within a conflict).
Each player i can exert effort $x_{ij} \in \mathbb{R}_+$ to increase her probability of winning the conflict against player j. We denote player i's action by $\mathbf{x_i} = (x_{ij})_{j \in N_i}$ which is a d_i-dimensional vector that contains all her effort choices.

The outcome of each bilateral conflict is determined by the total amount of efforts spent on that specific conflict. Player i's probability of winning is determined by a *contest success function* (from hereon CSF) $p(a_i x_{ij}, a_j x_{ji})$, where $a_i \geq 1$ captures how efficiently player i can employ her resources to increase this probability.[4] We use the same CSF as in the previous chapter as axiomatised by Skaperdas (1996).

$$p_{ij} = p(a_i x_{ij}, a_j x_{ji}) = \begin{cases} \frac{f(a_i x_{ij})}{f(a_i x_{ij}) + f(a_j x_{ji})} & \text{if } (x_{ij} + x_{ji}) \neq 0 \\ \frac{1}{2} & \text{if } (x_{ij} + x_{ji}) = 0 \end{cases} \tag{CSF}$$

Remember that this function is assumed to be increasing and concave in x_{ij} and decreasing and convex in x_{ji}. For this to hold, the impact function $f(.)$ is a positive and strictly increasing function of its argument with $f(0) = 0$ and is at least twice differentiable.[5]

Again, let $\boldsymbol{\omega} = (\boldsymbol{v}, \boldsymbol{a})$ be the combination of b values collected in $\boldsymbol{v}$ and the n efficiencies collected in $\boldsymbol{a}$. The space of all such combinations is $\Omega \subseteq \mathbb{R}^{b+n}_{++}$. A parametrisation in which $v_{ij} = v_{ji}$ is called *symmetric*. This is independent of the a_is. A set of parameters where all valuations across players and conflicts, and all efficiencies across all players are the same, a *strictly symmetric* parameterisation and denote it $\overline{\boldsymbol{\omega}} \in \Omega$.[6]

The difference to the previous chapter is that there are no direct costs of effort and players face a strict budget constraint over all the conflicts they are involved in.[7] This budget is denoted $R_i = R > 0$ and assumed to be symmetric across players.[8]

The payoff function is thus simply the sum of values of all conflicts weighted by their

[4] Think of this as the ratio of efficiencies relative to the weakest player w, given by $\frac{a_i}{a_w} \geq 1$. This assumption avoids problems with typical cost functions later. For most of our results it is not essential.

[5] The requirement of concavity for the CSF translates into the following condition on $f(.)$: $f''(a_i x_{ij})(f(a_i x_{ij}) + f(a_j x_{ji})) - 2f'(a_i x_{ij}) < 0$.

[6] This is a slight abuse of notation. In fact $\overline{\omega}$ is the whole set $\{(\lambda_1 \mathbf{1}_b, \lambda_2 \mathbf{1}_n) | (\lambda_1, \lambda_2) \in \mathbb{R}^2_{++}\}$, where $\mathbf{1}_k$ is a $k \times 1$ vector containing only ones. Our results hold for any element of this set.

[7] Since the marginal returns on each conflict are strictly increasing in own allocated resources, each player exerts the total amount $X_i = R$ across all conflicts even when constraint is not strict. We assume this here in the setup of the model to avoid the full characterisation of the best response function for the strategically irrelevant case when opponents choose $X_i < R$. See Kovenock and Rojo-Arjona (2019) for a discussion.

[8] Introducing asymmetry in this parameter is conceptually straightforward but does not change the model strategically. Defining all strategies as $\tilde{x}_{ij} = a_i x_{ij}$, the budgets in our model become $\tilde{R}_i = \frac{R}{a_i}$. Thus, different budgets are one interpretation of differences in efficiencies in this model.

respective probabilities of winning.

$$\Pi_i(\mathbf{x_i}, \mathbf{x_{-i}}, \mathcal{G}) = \sum_{j \in N_i} \pi_{ij} = \sum_{j \in N_i} p_{ij} v_{ij}$$

2.3 Equilibrium Analysis

The maximisation problem in this model is given by

$$\max_{\{x_{ij}\}_{j \in N_i}} \Pi_i(\mathbf{x_i}, \mathbf{x_{-i}}, \mathcal{G}) \quad \text{s.th.} \sum_{j \in N_i} x_{ij} = R.$$

There are no direct costs. Thus, increasing the effort level on one conflict only induces opportunity costs from reducing effort in another conflict. These opportunity costs across conflicts are mediated solely through the curvature of the CSF. The optimal amount of effort for each player $i \in \mathcal{I}$ for every pair of conflicts (ij) and (ik) is characterised by

$$\frac{v_{ik}}{v_{ij}} = \frac{f'(a_i x_{ij}) f(a_j x_{ji})}{f'(a_i x_{ik}) f(a_k x_{ki})} \frac{(f(a_i x_{ik}) + f(a_k x_{ki}))^2}{(f(a_i x_{ij}) + f(a_j x_{ji}))^2}.$$

We see that the level of valuations a player holds for different conflicts, say, a common factor c, would cancel. Only relative valuations matter when deciding how to distribute the fixed amount of total effort across conflicts. As in the previous chapter, players' marginal benefits are shaped by the effort exerted by their direct rivals but also their rivals' rivals and so on. This interdependency, induced by the cost function, determines how an individual reacts to changes in some of the parameters of the environment. Some of the behavioural implications thus occur even though a player's preferences (here meaning valuations) are not directly altered. Our result of impossibility to derive the equilibrium in closed form stated in the previous chapter and Cortes-Corrales and Gorny (2018), is valid in this setting.

As players cannot increase or decrease their total efforts X_i to any other level than R, players might have a problem equalising marginal revenues across conflicts. To see what that implies about interiority, consider Figure 2.3.1 . Let $M\Pi_i$ denote player i's marginal payoff. A player i, for whom $d_i = 2$, can pin down her decision on both conflicts by choosing only x_{i1} since $x_{i2} = R - x_{i1}$. The figure shows her marginal payoffs on both conflicts. Since they are both decreasing in the respective effort, the marginal payoff on conflict 2 is increasing in x_{i1}. With $M\Pi_{i1}$ (solid downward-sloping line), player i has enough endowment to balance her marginal payoffs. She can reach the intersection of the two functions within her budget constraint. Suppose we increase the conflict value from v_{i1} to $v'_{i1} > v_{i1}$. This results in the dotted line $M\Pi'_{i1}$. Now, it would be optimal to spend all resources on the conflict with the higher marginal revenue. That would seemingly result in a corner solution. But

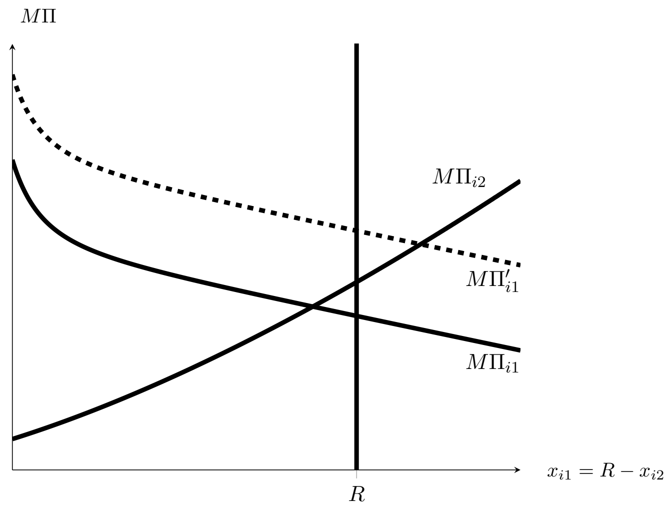

Figure 2.3.1 : Marginal Payoffs for a Player with 2 Conflicts
Note: The solid line at $x_{i1} = R$ represents the budget constraint. The solid curves represent the marginal payoffs before increasing v_{i2}. The dotted line shows the marginal payoff of conflict 2 after the change.

since an effort of 0 on the other conflict cannot be an equilibrium (we use the same CSF as in the previous chapter), an equilibrium, if it exists, can only be in mixed strategies.[9] We want to avoid running into this problem to analyse pure strategies and apply similar techniques as in Chapter 1. Note that this is not only a restriction on R alone. The marginal payoffs increase in the valuations and efficiencies, this also restricts the relative values $\frac{v_{i1}}{v_{i2}}$ and efficiency parameters (i.e., a_i and a_j). This mutual restraint can be characterised more precisely.

Essentially, we want to make sure that any pair of two marginal payoffs intersects for any effort choice on the interval $[0, R]$. Take some player $i \in \mathcal{I}$, and denote the highest valuation she holds with $v_{ih} = \max\{v_{ij} | j \in N_i\}$. The highest marginal payoff that player i can get in the conflict against h when $x_{ih} = R$ is

$$\max_{x_{hi} \leq R} \frac{\partial \Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}, \mathcal{G})}{\partial x_{ih}}\Bigg|_{x_{ih}=R} = \begin{cases} \frac{1}{4}\frac{a_i f'(a_i R)}{f(a_i R)} v_{ih} & \text{if} \quad \frac{a_i}{a_h} \leq 1 \\ \frac{a_i f'(a_i R) f(a_h R)}{(f(a_i R) + f(a_h R))^2} v_{ih} & \text{if} \quad \frac{a_i}{a_h} > 1 \end{cases}.$$

Since the first order condition of this problem is given by

$$\frac{\partial \Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}, \mathcal{G})}{\partial x_{ih} \partial x_{hi}} = \frac{a_i a_h f'(a_i x_{ih}) f'(a_h x_{hi})(f(a_i x_{ih}) - f(a_h x_{hi}))}{(f(a_i x_{ih}) + f(a_h x_{hi}))^3} \stackrel{!}{=} 0,$$

the argument that solves it is $x_{hi} = \frac{a_i}{a_h} x_{ih}$, meaning that the highest marginal payoff for a specific conflict is achieved when winning probabilities are the same for both players.

Equally well, on the conflict where i holds the lowest valuation $v_{il} = \min\{v_{ij} | j \in N_i\}$, we

[9] Whether it would be a strict mix of strategies for all players or only for a set of players is not clear though.

can find the minimal marginal payoff under any profile where $x_{ih} = R$ as

$$\min_{x_{li}\leq R} \frac{\partial \Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}, \mathcal{G})}{\partial x_{il}}\bigg|_{x_{il}=0} = \frac{a_i f'(0)}{f(a_l R)} v_{il}$$

When this marginal payoff (denote it $M\Pi_{ij}$ for some conflict (ij)) is larger than the one on (ih) when $x_{hi} = \min\{\frac{a_i}{a_h} x_{ih}, R\} = \min\{\frac{a_i}{a_h} R, R\}$, then player i has an incentive to divert resources to other conflicts, since $M\Pi_{iq} \geq M\Pi_{il}$ under this profile for $q \neq h$. Thus, we want to assert that

$$\min_{x_{li}\leq R} \frac{\partial \Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}, \mathcal{G})}{\partial x_{il}}\bigg|_{x_{il}=0} > \max_{x_{hi}\leq R} \frac{\partial \Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}, \mathcal{G})}{\partial x_{ih}}\bigg|_{x_{ih}=R}$$

To complete this argument, one has to notice that for $x_{ih} = 0$ and $x_{hi} = R$, we have $M\Pi_{ih} \geq M\Pi_{iq}$ for all $k \neq h$. This implies that each player can achieve $M\Pi_{ij} = M\Pi_{iq}$ for all $\{j, k\} \in N_i$. If that holds, we can use the same techniques as in the former proof to obtain the following result.

Proposition 2.1.
A set of locally unique, interior, pure-strategy Nash equilibrium exists for any $\boldsymbol{\omega} \in \Omega$, which is such that for all $i \in \mathcal{I}$ we have

$$\frac{v_{il}}{v_{ih}} > \frac{1}{4} \frac{f(a_l R)}{f(a_i R)} \frac{f'(a_i R)}{f'(0)}. \tag{2.1}$$

The solution function $x(\boldsymbol{\omega}) : \Omega \mapsto \mathbb{R}^{2b}_{++}$, mapping any parameter $\boldsymbol{\omega}$ into a Nash equilibrium $x(\boldsymbol{\omega})$, is at least C^2 and its derivative is given by

$$D_x(\boldsymbol{\omega}) = -\left[D_x F(x(\boldsymbol{\omega}); \boldsymbol{\omega})\right]^{-1} D_{\boldsymbol{\omega}} F(x(\boldsymbol{\omega}); \boldsymbol{\omega}). \tag{2.2}$$

There exists an open neighbourhood around any strictly symmetric $\boldsymbol{\omega} \in \Omega$, such that the corresponding equilibrium is globally unique.

Therefore, the interplay of conflict values, the initial endowment and properties of the impact function $f(.)$ determine whether the game has equilibria with the stated properties.

This condition is also interesting in light of a result by Friedman (1958) and its critical appraisal in Kovenock and Rojo-Arjona (2019). The latter provides an example in which the best response functions obtained in Friedman (1958) result in negative efforts and give a full characterisation of the best responses with zero efforts for certain distributions of the opponent's efforts. The difference to our model is, that the total amount of all effort against player i (across all her conflicts), can be as large as $d_i R$, while in a Colonel Blotto game it is limited to R, since all conflicts are shared with the same opponent. For a pure strategy equilibrium, this cannot be relevant. A conflict with zero efforts either, in case of exactly one player exerting zero effort, requires a 'smallest effort' which does not exist, or both players have a profitable deviation, when both of them exert zero effort. Our condition is essentially ruling out, that a best response of zero is strategically relevant.[10]

Two extreme cases for the assumptions on $f(x)$ come to mind. If we assume that the impact of the first marginal unit of effort is arbitrarily large, that is $\lim_{x\to 0} f'(x) = \infty$, there is no bound on budgets and valuations anymore. The canonical case of $f(x) = x^r$ for $r \in (0,1)$ has this property.
This immediately also points at a limitation of the proposition as it provides an empty set of parameters for existence and uniqueness whenever $f'(0)$ is arbitrarily small or even 0. The case $f(x) = x^r$ for $r \in (0,2)$ satisfies this and we cannot provide an insight in models that use this functional assumption. To restate this more clearly, the above result provides a sufficient condition for the existence of this type of equilibria, but examples where it is not necessary do exist.

For the rest of the chapter we shall assume that $f'(0)$ is sufficiently bounded away from 0 such that it provides a possible choice of budgets, valuations and efficiencies that guarantee that the model is well-behaved.[11]
The qualitative implications for k-regular networks – a network in which every player $i \in \mathcal{I}$ has $d_i = k > 0$ – found in the previous chapter and Cortes-Corrales and Gorny (2018), all go through in this setting as long as the above condition is met. More specifically we have the following.[12]

Proposition 2.2.
In a k-regular network the partial derivatives around the equilibrium at an arbitrary strictly symmetric parametrisation $\boldsymbol{\omega} \in \Omega$ can be obtained analytically as

$$\begin{aligned} \frac{\partial x_{ij}}{\partial v_{ij}} &= -\frac{(k-1)}{k}\frac{\overline{a}p^1}{z} > 0 \\ \frac{\partial x_{il}}{\partial v_{ij}} &= \frac{1}{k}\frac{\overline{a}p^1}{z} < 0 \quad \textit{for all } l \neq j \\ \frac{\partial x_{ij}}{\partial a_i} &= 0 \end{aligned} \tag{2.3}$$

for all $i \in \mathcal{I}$ and $(ij) \in B$ where $z = \overline{a}^2 p^{11} \overline{v}$. All other partial derivatives vanish.

Note that this result also obtains from taking the derivatives in Proposition 2 of Cortes-Corrales and Gorny (2018) for $C''(X) \to \infty$. This gives rise to a cost function that allows to

[10] Applying the same reasoning as above to the Blotto Game, in fact results in $\frac{v_{il}}{v_{ih}} > \frac{d_i - 1}{4}$.

[11] In fact from $\frac{v_{il}}{v_{ih}} \leq 1$ due to the definition of those valuations, we know that we need to require $f'(0) > \frac{1}{4}\frac{f(a_l R)}{f(a_i R)} f'(a_i R)$ for all i.

[12] We omit the proof here as it is almost identical to the one of Proposition 1.1.

account for both cases without assuming a discontinuous setting as in Kovenock and Roberson (2012). The following is an example of a cost function with $\rho > 1$ that can accommodate both cases by allowing $c''(X_i)$ to approach infinity as X_i approaches R.

$$c(X_i) = \frac{1}{\rho}\left(\frac{X_i}{R}\right)^{\rho} \tag{2.4}$$

Consider Figure 2.3.2 . For any $X_i < R$, the level of costs and the marginal costs get closer to 0, while large $X_i > R$ lead to large $c(X_i)$ and $c'(X_i)$. As $\rho \to \infty$ we arrive at the budget constrained setting discussed in this chapter. While for arbitrarily large levels of convexity

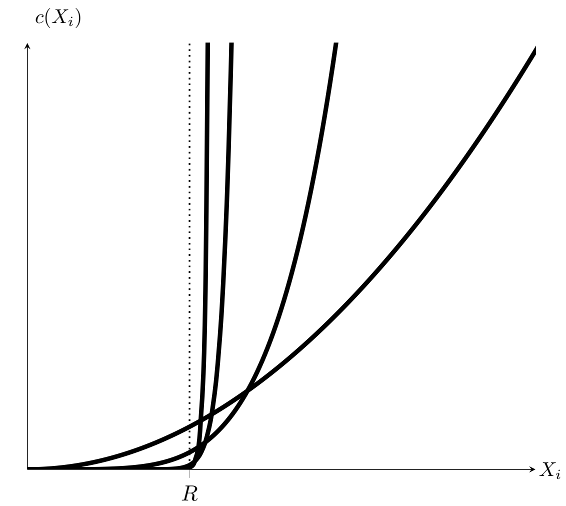

Figure 2.3.2 : Different Degrees of Cost Convexity

we have the same results as in the previous chapter and Cortes-Corrales and Gorny (2018). Some of these results change qualitatively when approaching infinity. The main benefit of this model becomes apparent when we turn to the classical case of a lottery contest success function with $f(x) = x$, as it allows us to describe each individual's behaviour given a set of opponents' effort levels.

2.4 Best Response Functions

The technical reason for a budget constraint is, that it provides one more degree of freedom when solving the first order conditions of each player. While this is slightly at odds with the finding that existence of finitely many, locally unique equilibria can fail as opposed to a setting with convex costs, it does hold for deriving individual best response functions.

The standard lottery CSF (Tullock, 1980), that obtains for $f(x) = x$ with equal splits to resolve ties, is given as

$$p_{ij} = p(a_i x_{ij}, a_j x_{ji}) = \begin{cases} \frac{a_i x_{ij}}{a_i x_{ij} + a_j x_{ji}} & \text{if } (x_{ij} + x_{ji}) \neq 0 \\ \frac{1}{2} & \text{if} (x_{ij} + x_{ji}) = 0. \end{cases}$$

We can now derive the best response functions that depend only on the characteristics of player i and her direct opponents $j \in N_i$.

Proposition 2.3.

With the CSF as defined in 2.5, the best response functions are given by

$$x_{ij}^{BR}(\{x_{li}\ \forall l \in N_i\}) = \frac{R_i + \sum_{m \neq i,j} x_{li}}{1 + A_{ij}} - \frac{x_{ji}}{1 + A_{ij}^{-1}} \tag{2.5}$$

for all $(ij) \in B$, *where* $A_{ij} := \sum_{l \neq i,j} \sqrt{\frac{v_{il}}{v_{ij}}} \sqrt{\frac{x_{li}}{x_{ji}}}$.

We suggest the following algorithm to solve the system of equations (for shorter notation we call this system x^{BR} here) given by Proposition 2.3.

Algorithm 1.

Choose some small $\overline{\delta} > 0$ *as your convergence criterion and proceed through the following steps.*

1. *Initialise vector* $x^0 \in \mathbb{R}^{2b}_{++}$
2. *Obtain* $x^{\tau+1} = x^{BR}(x^\tau)$
3. *Compute* $\delta = |x^{\tau+1} - x^\tau|$
4. *If* $\delta > \overline{\delta}$, *continue with step 2.*
5. *If* $\delta \leq \overline{\delta}$, *stop and return* $x^{\tau+1}$ *as the solution.*

We test this algorithm under the condition stated in proposition 2.1 for a canonical set of functional assumptions.[13]

Numerical Observation 1.

We fix $R \sum_{i \in \mathcal{I}} a_i = 1000$, $\sum_{(ij) \in B} v_{ij} + v_{ji} = 8000$ *and allow for* $v_{ij} \in [0.5\underline{v}, 1.5\underline{v}]$ *with* $\underline{v} = \frac{8000}{2b}$ *and* $n \in \{3, ..., 50\}$. *Using 10,000 random draws of parameters (a_i's, v_{ij}'s, n and $\mathcal{G}$'s)*[14] *that fulfil the above properties and condition* (2.1), *as well as 10,000 random draws for the initial vector* x^0, *Algorithm 1 never failed to converge.*[15]

[13] This exercise was done with MATLAB (2018) and code is available from the authors upon request.

[14] The latter was done by rounding uniform random matrices on $[0, 1]$ to the nearest integer and setting the diagonal elements to 0's, which provides us with an adjacency matrix.

[15] As an example what happens outside of condition (2.1), consider a complete network with $n = 3$, $R = 1$, $a_i = 1$ for all i and choose $v_{13} = 8$ and $v_{ij} = 1$ for all $(ij) \neq (13)$. We get the following cycle of iterations for any $x^0 >> 0$.

$$\begin{pmatrix} & 0.07 & 0.93 \\ 0 & & 1 \\ 0.467 & 0.533 & \end{pmatrix} \rightleftarrows \begin{pmatrix} & 0 & 1 \\ 0.356 & & 0.644 \\ 0.508 & 0.492 & \end{pmatrix}$$

One can speed this algorithm up by also considering the value of the normalised first order conditions at each x^N.

2.5 Revisiting Earlier Results

In Chapter 1 we conducted comparative statics around the symmetric parametrisation. With the algorithm at hand we can test some of these Propositions that are stated in terms of open neighbourhoods. A natural question that emerges is how large these neighbourhoods are and whether results carry over to stronger forms of asymmetry.
Let us restate Proposition 1.7 here before we investigate its robustness numerically.

Proposition 2.4 (Proposition 1.7)**.**
Fix a k-regular network and some $i, j \in S$. Let $\boldsymbol{\omega}' = (\overline{v}, ..., v_{ij} = \overline{v} + \epsilon, ..., \overline{v}, \overline{a}, ..., \overline{a})$. Furthermore, let $\Delta x_{lq} = x_{lq}(\boldsymbol{\omega}') - x^s$ for any $l, q \in S$. There exists some $\epsilon > 0$ such that the equilibrium $x(\boldsymbol{\omega}')$ for all $h, m \in S \backslash \{i, j\}$ satisfies

$$\Delta x_{ij} > \Delta x_{jh}, \Delta x_{hj}, \Delta x_{hm} > 0 > \Delta x_{ji}, \Delta x_{hi} > \Delta x_{ih} \tag{2.6}$$

Consider Figure 2.5.3 . On the y-axis we see the ratio of player i's valuation versus the symmetric valuations of all players $-i$. The lowest value shown here is 1, i.e., the strictly symmetric case, and the highest value is 4, which is the threshold according to Proposition 2.1. The dark grey area are the combinations of valuations and numbers of players n in the model, that result in equilibrium effort levels that are consistent with the inequalities in Proposition 1.7. The light grey are represents the area where at least one of them is not fulfilled.

For low numbers of players, the numerical investigation suggests that Proposition 1.7 can apply to the entire range of parameters for which an equilibrium is guaranteed to exist. As the number of players increases, higher levels of asymmetry cause the result to fail.
For efficiencies the previous chapter provided the result that if there is a stronger player ($a_i > a_{-i} = \overline{a}$), around the symmetric equilibrium, the remaining players decrease their efforts against the strong player and fight each other more fiercely. Again, we restate the proposition here for expositional purposes.

Proposition 2.5 (Budget Constraint Version of Proposition 1.6)**.**
Fix a k-regular network and some $i \in S$. Let $\boldsymbol{\omega}' = (\overline{v}, ..., \overline{v}, \overline{a}, ..., a_i = \overline{a} + \epsilon, ..., \overline{a})$. Furthermore, let $\Delta x_{lq} = x_{lq}(\boldsymbol{\omega}') - x^s$ for any $l, q \in S$. There exists some $\epsilon > 0$ such that the equilibrium $x(\boldsymbol{\omega}')$ for

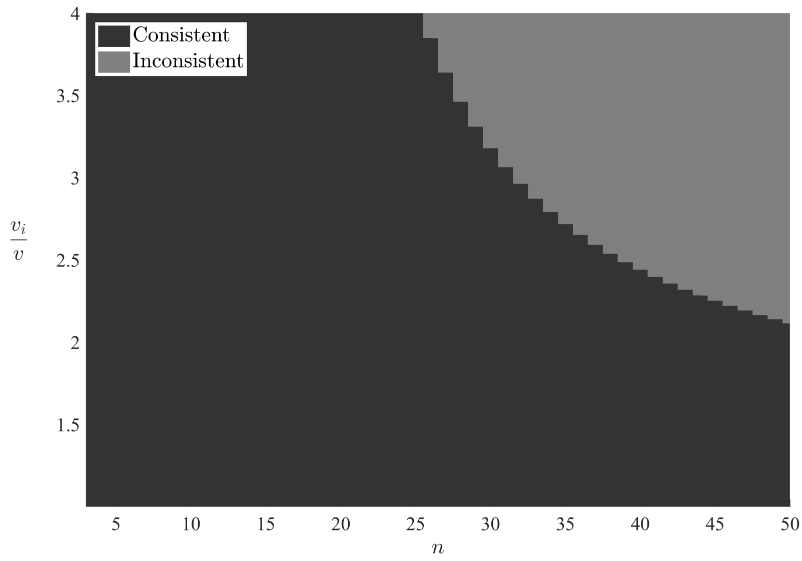

Figure 2.5.3 : Areas in which Proposition 1.7 Holds or Fails
Note: For each n we computed the equilibrium for 5000 evenly spaced values for $\frac{v_i}{v}$ in the segment $[0, 4]$.

all $j, m \in S \backslash \{i\}$ *satisfies*

$$\Delta x_{jm} > 0 > \Delta x_{ji}$$

Due to expression (2.3)*, we have* $\Delta x_{ij} = 0$.

Note that the change in efforts for players $-i$ is solely induced by the change of *effective efforts* ($a_i x_{ij}$) of player i, since with the budget constraint $x^s = \frac{1}{d_i} R$ for all $j \in N_i$ cannot change.

The corresponding graph to Figure 2.5.3 is omitted since it shows a plain dark grey area for all parameters we tested.[16] It seems thus to be fairly easy to find a wider range of parameters for which Proposition 1.6 holds that are further away from the strictly symmetric parameterisation.

2.6 A Few Further Interesting Examples

Two types of results are yet hard to be established in general. One concerns the effect that the degree d_i has on the players' strategies, particularly when these are asymmetric. Another question is how asymmetry can affect payoffs compared to players that are symmetric to one another. The following sections shed light on this by employing the above algorithm to two interesting examples.

[16] Again, we increased only player i's efficiency starting from $\frac{1}{4}\bar{a}$ while holding $\bar{a}$ constant for all $-i$. According to Proposition 2.1 the upper boundary for a_i is 4 in this case as well.

2.6.1 The Enemy of My Enemy Is My Friend: Centrality as a Determinant of Strength

Suppose all valuations, efficiencies and budgets are equal to 1. To have a benchmark case consider the two disjoint networks in Figure 2.6.4 . The number next to each player indicates

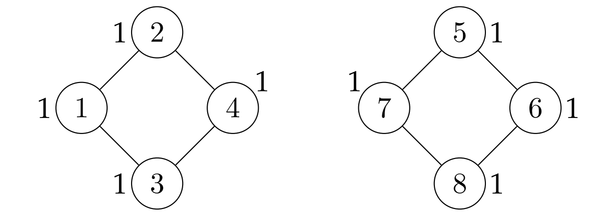

Figure 2.6.4 : Two Disjoint 4-Player Ring Networks

Note: The graph shows payoffs in equilibrium for $\overline{v} = 1$, $\overline{a} = 1$ and $R = 1$ outside of the nodes.

the level of payoff she is receiving in equilibrium. The payoffs are equal to 1 here since every player receives the prize of value 1 with probability $\frac{1}{2}$ on two conflicts.

Now suppose we link the two players from both networks whose nodes are closest to each other, i.e., players 4 and 7. The increase in payoffs for the connected players is mainly due to

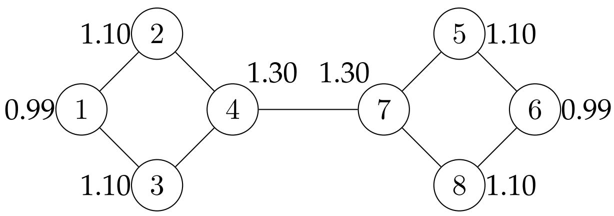

Figure 2.6.5 : Ring-Player 4 versus Ring-Player 7

the additional prize that they can win. If we adjust this payoff with the number of conflicts, we see that on average, these players have a lower expected payoff per conflict.

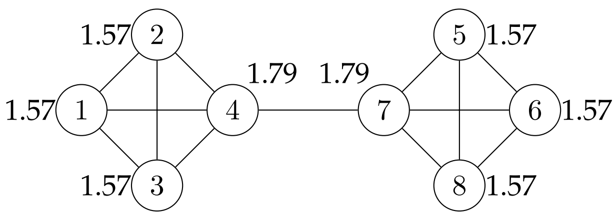

Figure 2.6.6 : Complete-Player 4 versus Complete-Player 7 Network

More interestingly, the payoffs of the two players on the outside (1 and 6) have reduced. Since the increase in player 4's and 7's opponents has made them weaker, our earlier intuition applies and players 2,3 and 5,8 attack the outside players 1 and 6 more fiercely, respectively. One could say that 'the enemies of my enemies' enemies' are also my enemies'.

The main reason for the outside players to lose from that change in the network is due to the fact that they cannot attack the relatively weak central players. If we change that by

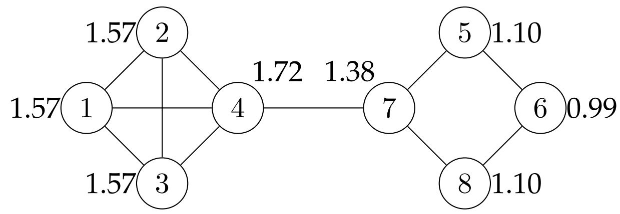

Figure 2.6.7 : Complete Player 4 versus Ring-Player 7 Network Player

making both sides of the network complete, we see that now players 4 and 7 are the only ones that loose on average ($\frac{1.79}{4} < \frac{1.57}{3}$).

2.6.2 Asymmetric versus Symmetric Teams

Consider a bipartite network with three players on each side as illustrated in figure 2.6.8 . Let all valuations be $\overline{v} = 1$ and let budgets be $R = 3$.[17] Team A has efficiencies $a_i = 4$ for $i \in \{1, 2, 3\}$ and team B has efficiencies $(a_4, a_5, a_6) = (6.5, 4, 1.5)$. Note that the condition in Proposition 2.1 is satisfied for these values. Each of the players from team B is facing

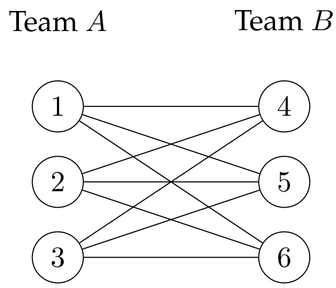

Figure 2.6.8 : Bipartite Network Structure: Team A vs. Team B

a set of homogenous players and thus their strategies are simply given by $(1, 1, 1)$, which is the equal split of their budget ($R = 3$) over their three conflicts. The behaviour for the homogenous players can be seen in Figure 2.6.9 . We use player 2 to exemplify the strategies of all players in team A, since they all face the same problem. As in our earlier results, we see that the efforts are higher, the more similar an opponent is with respect to efficiencies, since $\frac{\overline{a}}{a_6} = \frac{64}{24} > \frac{a_4}{\overline{a}} = \frac{39}{24} > \frac{a_5}{\overline{a}} = 1$.[18]
Calculating the payoffs from the these effort levels we find

$$\Pi^A = \sum_{i=1}^{3} \Pi_i < \sum_{i=4}^{6} \Pi_i = \Pi^B$$

[17]Since the symmetric equilibrium is $\frac{1}{d_i}R$, these numbers provide better numbers in terms of presentation. Any other parametrisation that is qualitatively the same has produced the same results in our experimentation with the algorithm.
[18]We compare $\max\{\frac{a_i}{\overline{a}}, \frac{\overline{a}}{a_i}\}$ for all $i \in \{4, 5, 6\}$ here, as only the factor of the ratio matters.

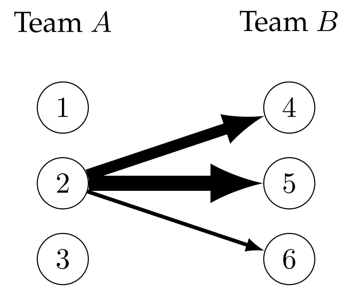

Figure 2.6.9 : Homogenous Team A vs. Heterogenous Team B

Thus, if there are two teams in which each player individually decides on the amount of effort to spend, asymmetries can help the team to perform better in total. Individually, the strong ($i = 4$) and the weak ($i = 6$) player have higher payoffs than all other players in the game. The payoffs of player 1 to 3 are naturally identical and greater than the medium strength player ($i = 5$) in the heterogenous team.

2.7 Conclusion

We have introduced budget constraints into a model of conflict networks. Whenever strategies are guaranteed to be interior, earlier results about existence and uniqueness apply. A sufficient condition is presented for when interiority holds. When the CSF rewards very small efforts sufficiently it is always satisfied. For a canonical case we provide an algorithm that seems to converge to the Nash equilibrium, whenever the initial vector of efforts is strictly positive and below each player's budget. This algorithm allowed us to investigate 'how large' the neighbourhoods around symmetric parameterisations are in which former results were obtained. We show that there exist a large number of cases were they do and provide further examples that could not yet be solved for theoretically. Two examples were considered numerically, using the algorithm, that could not be studied before theoretically within this class of models.

This chapter can give a foundation for two-stage models, in which the military budget is decided in stage one and the military interaction takes place in stage two. This can extend papers like Neary (1997), who have analysed such dynamic questions in a simpler framework that does not account for network effects of more distant conflicts. Since techniques like backward induction rely on stage payoffs, the algorithm provided allows to conduct such an analysis.

More generally, as the baseline model with convex costs is technically more intractable, any results based on the analysis provided here can provide the field with intuition about the general class of models to then be theoretically investigated.

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2.A Proofs

Proof of Proposition 2.1

The steps in the main text have established that for any choice of $\boldsymbol{\omega}$ that fulfils the condition provided in the proposition, the solution is described by the FOCs of the maximisation problem.

Existence follows from the assertions. The game has a finite set of player and continuous payoff functions, when employing the given CSF. These payoff functions are strictly concave. Thus, it is given by the system of first order conditions of the Lagrangian given by

$$\mathcal{L}\left(\{x_{ij}\}_{j\neq i}, \lambda_i, \mathcal{G}\right) = \pi_i(\{x_{ij}\}_{j\neq i}, \{x_{ji}\}_{j\neq i}) + \lambda_i \left(R_i - \sum_{j\neq i} x_{ij} \right),$$

which obtain as

$$\begin{aligned} F_{ij} &= a_i p_{ij}^1 v_{ij} - \lambda \stackrel{!}{=} 0 \;\; \forall j \neq i && (d_i \times M\Pi_{ij}) \\ F_{ii} &= R_i - \sum_{j\neq i} x_{ij} \stackrel{!}{=} 0, && \text{(Budget)} \end{aligned}$$

for all $i \in \mathcal{I}$.[19]

The entire system of equations is denoted by $F = (F_{ij}|(ij) \in B)$. To complete the proof we want to verify that the determinant of the Jacobian $H = D_{\boldsymbol{x}}\boldsymbol{F}$ of this system is positive, with altering signs for its leading principal minors, irrespective of the choice of $\boldsymbol{\omega}$. This establishes negative definiteness of H and thus concavity of payoffs.

$$\det(H) > 0 \quad \forall \boldsymbol{\omega} \in \Omega.$$

The determinant of the Hessian of player i is given by

$$H_i = \begin{pmatrix} a_i^2 p_{i1}^{11} v_{i1} & & & & & -1 \\ & \ddots & & & & -1 \\ & & a_i^2 p_{ij}^{11} v_{ij} & & & -1 \\ & & & \ddots & & -1 \\ & & & & a_i^2 p_{id_i}^{11} v_{id_i} & -1 \\ -1 & -1 & -1 & -1 & -1 & 0 \end{pmatrix}$$

[19]Denoting the budget constraint as F_{ii} is a slight abuse of notation, as this does not refer to the conflict of player i against herself. We merely use it as it is consistent with the other notation and 'available' as a shorthand.

All the entries not shown here are zero. Due to this and the zero entry in the bottom right corner (the derivative of the budget constraint w.r.t. λ_i) there is no permutation greater than one that leaves a nonzero diagonal product. Thus the determinant of this matrix is given by

$$\det(H_i) = -\sum_{j \in N_i} \prod_{q \in N_i \setminus \{j\}} a_i^2 p_{iq}^{11} v_{iq}$$

which is negative whenever d_i is even and positive whenever d_i is odd. Deleting the last row and column, the determinant of that new matrix is simply given by

$$\prod_{j \in N_i} a_i^2 p_{ij}^{11} v_{ij},$$

which is positive whenever d_i is even, and negative whenever d_i is odd. All the remaining leading principal minors obtain from removing one $a_i^2 p_{ij}^{11} v_{ij}$ from the product. Thus the sign alternates and we conclude that H_i is negative definite and thus the payoff functions are strictly concave.

The matrix H than obtains as a block matrix that has all H_i on the diagonal and all off-diagonal blocks are identical to expression (1.10) from the previous chapter. Since there is no negative summand coming from a second derivative of a cost function, the same logic as there applies and we can conclude that $\det(H) > 0$ for all $\boldsymbol{\omega} \in \Omega$.

Finally, note that the function given by Rosen (1965), obtains as

$$\sigma(\boldsymbol{x}, \boldsymbol{r}) = r \sum_{i \in \mathcal{I}} \pi_i = r \sum_{i \in \mathcal{I}} p_{ij} v_{ij}, \tag{2.7}$$

which is concave, by the concavity of the individual payoff functions.

All the remaining steps are identical to the proof of Proposition 1.1.

Proof of Proposition 2.3

Equating any two of the d_i first order conditions derived in the preceding proof, say q and m yields

$$\begin{aligned} \frac{x_{qi}}{x_{mi}} &= \left(\frac{x_{qi} + x_{iq}}{x_{mi} + x_{im}}\right)^2 \frac{v_{im}}{v_{iq}} \\ \Leftrightarrow x_{im} &= \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}} (x_{qi} + x_{iq}) - x_{mi} \end{aligned}$$

Now we sum up over all $m \neq i, q$ to arrive at the following expression, using the budget constraint.

$$R_i - x_{iq} = \left(\sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}} \right) (x_{qi} + x_{iq}) - \sum_{m \neq i,q} x_{mi}$$
$$\Leftrightarrow \left(1 + \sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}} \right) x_{iq} = R_i + \sum_{m \neq i,q} x_{mi} - \left(\sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}} \right) x_{qi}$$

Dividing by the factor in parenthesis that precedes x_{iq} and noticing that all maximisation problems are symmetric (though not identical) we arrive at the $2b$ best response functions for each player against each opponent.

$$\begin{aligned} x_{iq} &= \frac{R_i + \sum_{m \neq i,q} x_{mi}}{1 + \sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}}} - \frac{\sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}}}{1 + \sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}}} x_{qi} \\ &= \frac{R_i + \sum_{m \neq i,q} x_{mi}}{1 + \sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}}} - \frac{x_{qi}}{1 + \left(\sum_{m \neq i,q} \sqrt{\frac{v_{im}}{v_{iq}}} \sqrt{\frac{x_{mi}}{x_{qi}}} \right)^{-1}} \end{aligned}$$

More precisely, defining $A := \sum_{m \neq i,j} \sqrt{\frac{v_{im}}{v_{ij}}} \sqrt{\frac{x_{mi}}{x_{ji}}}$, we have the desired result.

2.B MatLab Code

MatLab Code 2.1: Script for Best Response Functions

```
        function [fx]=bestresponses(x,v,R,n,a)

%In this function x is the matrix of efforts,it must be a nxn matrix with
%zero diagonal, v is the matrix of efforts (think of this as a weighted
%adjacency matrix, thus v_ij=v_ji=0 means that players do not have a conflict)
%must be a nxn matrix with zero diagonal, R is the budget, it must be a
%positive scalar, n is the Number of Players, a is the vector of the n
%efficiencies. The output fx is the (nxn) matrix of best responses for the
%(nxn) input of efforts in x.
```

```

fx=zeros(n,n-1);

for i=1:n
    for j=1:n
        if j==i
            fx(i,j)=0;
        else
        fx(i,j)=(R(i,1)+1/a(i)*(sum(a.*x(:,i))-a(j)*x(j,i)))
           /(1+(1/(sqrt(a(j)*v(i,j))*sqrt(x(j,i))))*(sqrt(a'.*v(i
           ,:))*sqrt(x(:,i))-sqrt(a(j)*v(i,j))*sqrt(x(j,i)))) -...
            ((a(j)/a(i)*x(j,i))/(1+((1/(sqrt(a(j)*v(i,j))*sqrt(x(j,
               i))))*(sqrt(a'.*v(i,:))*sqrt(x(:,i))-sqrt(a(j)*v(i,j
               ))*sqrt(x(j,i))))^(-1)));
        end
    end
end

for i1=1:n %This part is only relevant if parameters are outside
   condition (2.1)
    for j1=1:n
        if fx(i1,j1)<0
            fx(i1,j1)=10^(-14);
        elseif fx(i1,j1)>R(i1,1)
            fx(i1,j1)=R(i1,1);
        end
    end
end


end
```

MatLab Code 2.1: Script for Best Response Functions

MatLab Code 2.2: Script for Algorithm 1 (for **b**udget **c**onstrained **n**etworks)

```
        function [x,ef]=bcnalgorithm(v,R,n,a,varargin)

%In this function x is the matrix of efforts,it must be a nxn
```

```
matrix with
%zero diagonal, v is the matrix of efforts (think of this as a weighted
%adjacency matrix, thus v_ij=v_ji=0 means that players do not have a conflict)
%must be a nxn matrix with zero diagonal, R is the budget, it must be a
%positive scalar, n is the Number of Players, a is the vector of the n
%efficiencies. The exit flag ef is 1 in case of convergence and 0
%otherwise. Initial vector for x_0 can be provided in varargin, default is a matrix of ones.


if isempty(varargin)==1
    x_in=ones(n,n);
    x_in(1:n+1:end) = 0;
else
    x_in=varargin{1};
end

iterstep=0;
d=10;
ef=0;

while d>=10^(-13) && iterstep<1000

    iterstep=iterstep+1;
    x_out=n3nbudgeteff(x_in,v,R,n,a);
    d=max(max(abs(x_in-x_out)));
    x_in=x_out;

end

if iterstep<1000
    ef=1;
end
```

```

x=x_in;

end
```

MatLab Code 2.2: Script for Algorithm 1 (for **b**udget **c**onstrained **n**etworks)

Part II

Social Preferences in Contests

Chapter 3

Social Preferences in Contests with Heterogenous Players[1]

[1]I would like to thank participants of the internal seminar at the University of East Anglia in general and Subhasish M. Chowdhury, David Hugh-Jones and Mich Tvede in specific for useful comments. I am especially thankful for help of Anwesha Mukherjee, who helped drafting an earlier version of this paper. All remaining errors are my own.

Paul M. Gorny[‡]

Abstract

We analyse models of social preferences in lottery contests. Players can either compare ex-ante expected payoffs/shares of total payoff or ex-post (consequentialist) payoffs conditional on winning or losing. Also, they might or might not take costs into consideration. We derive the best response functions for all four resulting models and the equilibria, whenever possible. We find that the ex-ante setting with cost comparison predicts lower total efforts than the setting without costs, when players are 'spiteful'. Both models can have continua of equilibria. We show that those are not robust to heterogenous perceptions of fairness. We present numerical evidence that overspreading of bids is less pronounced under the ex-ante formulation. Incorporating costs in the ex-post model increases total efforts, but excluding them makes the model the most tractable presented here. The strong player might exert the highest effort when costs are asymmetric, which is at odds with the standard model. This can have implications for the design of contests when effort is desirable. We critically assess previous attempts to implement the models with experimental data.

JEL: C72, D84

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3.1 Introduction

When fighting or competing with others, we have a sense of how deserving our opponents are. If they tried hard it is easier to accept to have lost, while a win of someone who, at least subjectively, did not deserve to win, is seen as unfair. This applies when sportspeople indicate that their opponents, or even themselves, did not deserve to win (see e.g. Dominic Thiem's assessment of not deserving ending up in the ATP semi-finals last year). If a co-worker is receiving the promotion one was after, this is particularly bitter, if a lot of work was put in.

While this post-competition behaviour is observable, the question remains, whether these considerations are already made when deciding on how much effort to exert in the competition in the first place. Do we typically internalise the costs our opponents have when determining what a fair outcome is? Or are we solely focussed on the distribution of the price? Since costs differ with the level of ability, this might affect efforts in different ways depending on whether we have these concerns or not.

Think of a two competing workers A and B in a company who seek to get promoted. Suppose A has a higher likelihood of getting the promotion but is also staying in the office a lot longer than B. Depending on how much longer A stays, B might consider the eventual promotion of A as unfair, as fair or even feel guilty for being considered in that competition at all, given her relatively low effort.

Another question is how much individuals acknowledge their position as favourite or underdog in the competition. Do they assume straightaway to win against a weaker opponent and base their considerations on the expectation to face advantageous inequality? Or do they weigh the possibilities of winning and losing? The former is a comparison based on an *ex-ante* expectation of the outcome and thus, once formed, easy to compare. Technically, if A expects to win £10 in a lottery and expects B to win £5, there is no scope for disadvantageous inequality concerns for A. In contrast, in the *ex-post* view, if A sees a certain chance of winning the lottery while B loses, and a certain chance that the opposite happens, A weighs the concerns for advantageous inequality (from hereon AI) and disadvantageous inequality (from hereon DI) with the respective probabilities. There are two sources for where this distinction can come from.

For probabilistic contests, the ex-post view is clearly the more conventional application of standard expected utility. The argument for an ex-ante formulation in such a game can come from the results under strong forms of cost asymmetry. While the ex-post setting allows for concerns for AI and DI to be present in both players, even if the strong player is clearly much more likely to experience AI rather than DI, the ex-ante models suggest that only the AI concern matters for the strong player. Consider another example. A professional chess player P playing against an amateur A. Both players are well aware of the P's advantage. The eventual behaviour under social preferences should exhibit concerns for AI for P and concerns for DI for A. The game they are playing is an 'unfair procedure' to determine a winner. Even though A is still most likely to lose the match, P might give A a chance. Thus, the players do not care about the outcome, which is the same (P wins and A loses). Rather, they care about how badly A loses against P and try to reduce this difference. There is evidence for this preference for procedural fairness, i.e., whether the procedure leading to a certain distribution of resources is fair (Konow, 2003).

A much clearer case can be made for share contests. In this type of contests, instead of winning the prize with a certain probability, players receive a share of the prize. This share is proportional to their effort. It is not possible that both players exert positive effort and one of them receives the full prize. Thus, the consequentialist payoffs from a probabilistic contest are meaningless in this framework. With standard preferences, the distinction between probabilistic and share contests is made in their interpretation. They are mathematically the same. When social preferences are invoked, this is no longer true (Fudenberg and Levine, 2012). This type of contest can be observed frequently in reality, e.g., in rent seeking, competition for market shares and litigation.

This paper contrasts the resulting four different model specifications of comparing revenues vs. net-payoffs and making ex-ante vs. ex-post comparisons, respectively. We discuss the standard theoretical properties of existence, equilibrium characteristics and comparative statics with respect to cost asymmetry. Further, we discuss a few implications some of these properties have for using social preferences in the analysis of experimental data. Typical phenomena in the lab are *overspending*, *overdissipation* and *overspreading*. Overspending refers to substantially higher bids than the the Nash equilibrium prediction of the standard model. These also do not converge to the Nash prediction over repeated interaction (Sheremeta, 2013). If it is very pronounced, subjects exert more total effort than the value of the prize, also known as overdissipation. Overspreading refers to bids varying widely across the strategy space, which cannot easily be explained by small decision errors (Chowdhury et al., 2014). The models presented here can shed light on some of this behaviour. We intend to highlight, which models fail to account for (parts of) them. As such, we also discuss the practical use of the models. A procedure that has been used in the literature, is found to be problematic when considering cost asymmetries. Consider Table 3.1. Two of the specifications we just sketched have already been studied in the literature. Comparing them in terms of the different notions of expectations and cost considerations, reveals that

Specification	Ex-Ante	Ex-Post
Revenues	Model AR (Hoffmann and Kolmar, 2017)	Model PR
Net Payoffs (Revenues - Costs)	Model AC	Model PC (Herrmann and Orzen, 2008)

Table 3.1: Different Models

Note: Names indicate whether model is ex-**A**nte or ex-**P**ost and whether **R**evenues are considered without or with **C**osts

moving between them alters two modelling choices at a time. When using a model of revenue comparison, only an ex-ante/share model is available. When choosing net-payoffs as the basis of social comparison, this is so far only modelled with ex-post comparison. For a proper discussion on how these aspects contribute to the analysis of behaviour in contest, theoretically and experimentally, Models AC and PR are needed.

In the seminal paper of Fehr and Schmidt (1999), an agent's utility is given by:

$$U_i(\pi_i, \pi_{-i}) = \pi_i - \alpha_i \max\{\pi_{-i} - \pi_i, 0\} - \beta_i \max\{\pi_i - \pi_{-i}, 0\} \tag{3.1}$$

where π_i is the material payoff to player i, the preference parameters for disadvantageous and advantageous inequality are respectively given by $\alpha_i > 0$ and $\beta_i \leq \alpha_i$. The term π_{-i} represents some summary measure of the payoff of the other player(s), typically the arithmetic mean.

In research with environments where cooperative elements are present, it is often assumed that β_i is non-negative (Anderson et al., 2002, See e.g.). This amounts to assuming that individuals have, at least to some extent, altruistic motives. An individual can have a detrimental contribution to her utility if she overreaches others in her surrounding. The behaviour of individuals in conflict differs systematically from these settings. Spite is the *enjoyment deriving from advantageous inequality*. Herrmann and Orzen (2008) suggest that it is present in contest settings in the lab. Formally, this amounts to assuming $\alpha_i \geq |\beta_i|$ while allowing for $\beta_i < 0$. We use the terms by Hoffmann and Kolmar (2017) and refer to an individual with

- ...$\alpha_i > 0$ and $\beta_i > 0$ as **inequality averse**,
- ...with $\alpha_i > 0$ and $\beta_i < 0$ as **inequality prone** or **spiteful** and
- ...with $\alpha = 0$ and $\beta = 0$ as **selfish**.

Conflict and competition are often formally modelled as contests, where players spend costly resources to improve their winning probabilities for a prize of given value. The net expected payoff to a contestant is the probability-weighted sum of his or her net payoffs in the mutually exclusive events of a win and a loss. While there are studies on how social preferences can be implemented on probabilistic games or decision problems (see Levitt and List (2007) for a survey of these), contests differ in an important aspect from other choice situations. The afore-mentioned studies focus on random events with fixed probabilities, that are exogenously given or determined by nature. In a probabilistic contests, both, the ex-post payoffs as well as the ex-ante probabilities of winning, are affected by both players' actions. Paired with the continuum of strategies, this aspect makes theoretical contests considerably more complex, when imposing social preferences.

Our study relates to the existing literature on models of and experiments on distributional preferences in contests. A number of studies (Grund and Sliwka, 2005; Herrmann and Orzen, 2008; Fonseca, 2009; Lim, 2010; Rockenbach and Waligora, 2016; Chowdhury et al., 2018) have shown that contest behaviour in the lab can be structured more precisely within the framework developed by Fehr and Schmidt (1999). Such distributional preferences have also been offered as one of the several explanations behind the widely reported phenomena of overspending, overspreading and overdissipation in contests. The majority of these studies assume that individuals maximise the probability-weighted sum of their ex-post utilities. That is, the expected total utility of a contestant with non-degenerate social preferences is the sum of the ex-post utilities she would obtain in case of a victory and a defeat, weighted by the respective probabilities.

This differs from fairness concerns as argued by Rabin (1993) or as implemented by Falk et al. (2008). Since our models are all in a setting of complete information there is no scope for beliefs, and thus for intentions that differ from actions.[2] Nonetheless, one can make a distinction of different types of justice. The ex-post specifications models individuals concerned with distributive justice. In the ex-ante individuals are concerned with procedural justice (Krawczyk, 2011). The study at hand should be understood as a first step in discriminating between the different approaches by characterising each of them in terms of their theoretical properties.

We find that in all models the equilibrium efforts strongly depend on the DI and AI parameters and their interplay with cost asymmetry. Including costs in the ex-ante specification results in similar qualitative implications to Hoffmann and Kolmar (2017) but overspreading is more pronounced for disadvantaged than for advantaged players. For these two models to exhibit continua of equilibria, players need to fully agree on what is fair. If players differ

[2] While the authors of the baseline model state that "[...] preference parameters are compatible with the interpretation of intentions-driven reciprocity." (Fehr and Schmidt, 1999, p.852), an argument also used by Hoffmann and Kolmar (2017), the fact that famous models capturing intentions have been developed by the same researchers leads us to believe that this model is rather about actions than intentions.

in the split they regard as fair, this result breaks down. While incorporating costs decreases total efforts in the unique equilibria of the ex-ante specifications, it can increase them in the ex-post specifications. In general, there is numerical evidence that the prediction for overspreading in Model AC is less pronounced than in Model AR. When comparing the different models we see that in all cases the stronger player might exert the highest effort for an unequal cost ratio. In the standard model, individual efforts are highest when players are identical with respect to the model parameters. This implies that some level of inequality in terms of costs can be increase the effort of the high-skilled player. This is particularly relevant for contests where effort is desirable, either because it is productive or because it serves as a signal of skill as in R&D contests (Che and Gale, 2003), public procurement contests (Dimitri et al., 2006) and contests for promotion (O'Keeffe et al., 1984).

The general study of social preferences in frameworks of competition is important for many applications in workplace competition, the design of sports contests and litigation. Whenever in such a setting a coworker, contestant or litigator faces a relatively high chance of winning, this might trigger behaviour reducing further efforts, in case of guilt, or even increasing it further in case of spite. The underdog in such a situation, might not stand a chance of winning. If she is prone to jealousy though, she might still increase efforts to lower her opponents chances or share of the pie.

The paper proceeds as follows. Section 3.2 discusses the models in the above table. Section 3.2.1 briefly summarises the key result of Hoffmann and Kolmar (2017). We spend more time elaborating the ex-ante model with net-payoff comparison in Section 3.2.2. We briefly digress on the continua of equilibria in these models in Section 3.2.3. There, we investigate the key underlying assumption of this result in the ex-ante models. In Section 3.2.2 we discuss Model PC, which is a variant for heterogenous costs of the model described in Herrmann and Orzen (2008). We contrast it with Model PR in Section 3.2.4 and investigate its properties. We summarise the differences and similarities between the models in Section 3.3 and discuss the effects of increased asymmetry between players on total efforts across the models in Section 3.3.1. Section 3.4 concludes.

3.2 The Models

Let there be two players $i \in \{1, 2\}$. They are simultaneously exerting effort $x_i \geq 0$, in order to win a prize of commonly known value $v > 0$. Without loss of generality, we assume player 1 to be the stronger player or the favourite, i.e., she has lower costs $c_2 > c_1$. This makes player 2 the weaker player or the underdog in this contest. We normalise the cost of player 1 to unity $c_1 = 1$. Thus, c_2 can be interpreted as the ratio of costs between the players and as the degree of asymmetry.

We use the contest success function for a simple lottery between two players (Tullock, 1980).

$$p_1(x_1, x_2) = \frac{x_1}{x_1 + x_2} = 1 - p_2(x_2, x_1) \tag{3.2}$$

In case of a tie, the prize is split equally among the players.[3] The difference between the four specifications lies in the interpretation of how players could form expectations about the final payoffs and whether players compare revenues or payoffs net of costs.
A couple of definitions are in order to keep the mathematical representation legible. For all $i \in \{1, 2\}$ and $i \neq j$, for the ex-ante models we define

$$\begin{aligned} u_i &= p_i v \\ w_i &= p_i v - c_i x_i \\ \Delta u_i &= u_i - u_j \\ \Delta w_i &= w_i - w_j \end{aligned} \tag{3.3}$$

while for the ex-post models, we define

$$\begin{aligned} y_i^W &= v \quad y_i^L = 0 \\ z_i^W &= v - c_i x_i \quad z_i^L = -c_i x_i \\ \Delta y_i &= y_i^W - y_j^L = v \\ \Delta z_i &= z_i^W - z_j^L \end{aligned} \tag{3.4}$$

Thus, Δu_i and Δw_i represent the comparison of the ex-ante expected payoffs, while Δy_i and Δz_i are comparing final payoffs conditional on winning, in each case either with or without incorporating the costs of effort.[4]

3.2.1 Model AR: Ex-Ante & Revenues (Hoffmann and Kolmar, 2017)

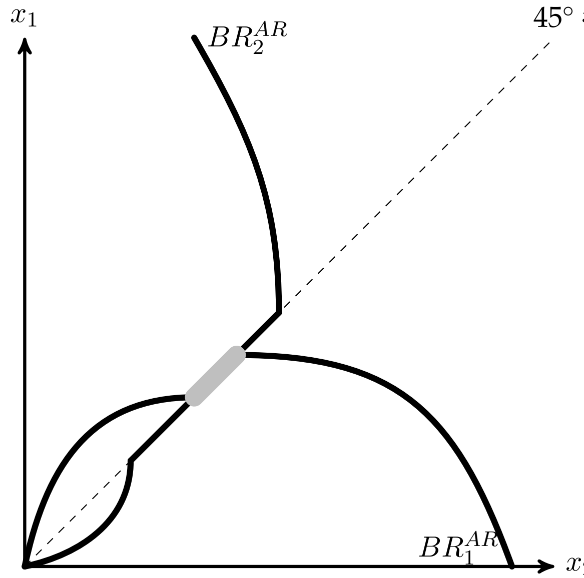
In this model, the utility function of player $i \in \{1, 2\}$ and $j \neq i$ is given by

$$U_i^{AR}(x_i, x_j) = w_i - \alpha_i \max\{-\Delta u_i, 0\} - \beta_i \max\{\Delta u_i, 0\} \tag{3.5}$$

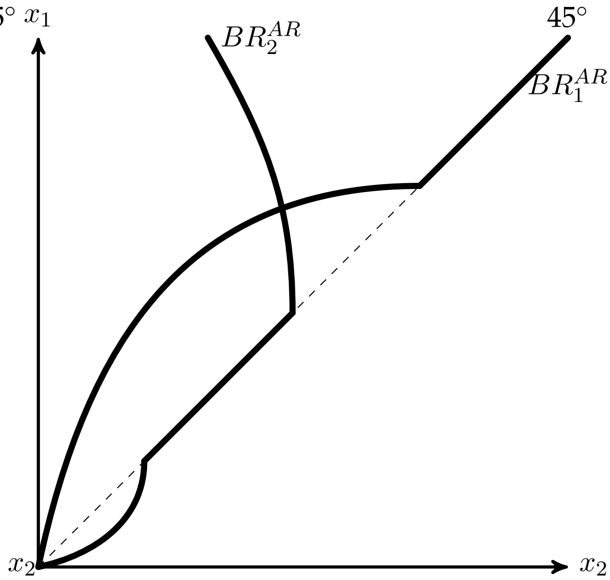
[3] This assumption is merely for completeness. All results go through as long as no player receives the prize with certainty at $(0, 0)$.

[4] In case of losing, the difference for player i is simply $-\Delta y_j$.

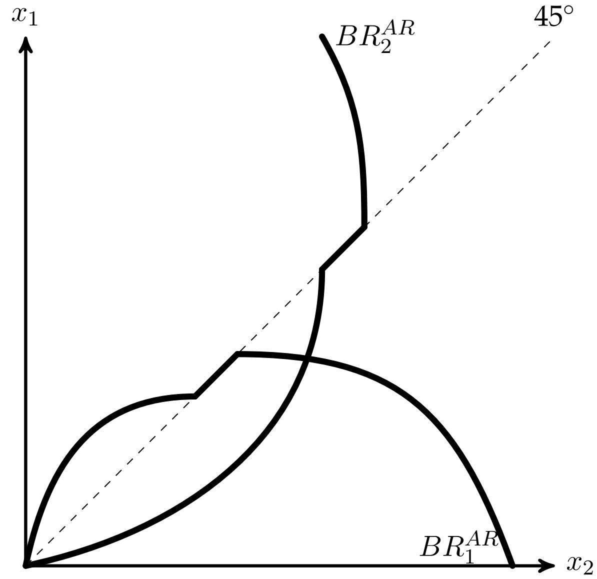
Focussing on the game with simultaneous moves and two players, the respective results of Hoffmann and Kolmar (2017) can be summarised by the following graphs and propositions. For ease of notation we define $a_i = 1 + 2\alpha_i$ and $b_i = 1 - 2\beta_i$.



(a) Two inequality averse players



(b) Two inequality averse players



(c) Inequality Prone Player 2

Figure 3.2.1 : Possible Equilibria

Note: All panels show the best response functions of both players as solid lines that have two concave parts (the one closer to the origin is the β-type's best response, the outer one is the α-types best response) and one linear part in between.In Panel (a) the best response functions overlap on their linear part. The resulting set of equilibria is indicated as the thick grey line. This is what we refer to as 'continuum of equilibria' in this model. Panel (b) shows a case where in equilibrium the underdog is jealous (she is her α-type) and the favourite feels guilty (she is her β-type). Panel (c) shows the reverse case, where, in equilibrium, the favourite is jealous (α-type) and the underdog is spiteful (β-type).
Source: Reproduced from Hoffmann and Kolmar (2017), Figures 1-3.

Lemma 3.1 (Hoffmann and Kolmar (2017)).

Player i's best response function is given by

$$BR_i^{AR}(x_j) = \begin{cases} \sqrt{b_i \frac{x_j}{c_i} v} - x_j & \text{for } x_j < x_j^\beta \\ x_j & \text{for } x_j \in \left[x_j^\beta, x_j^\alpha\right] \\ \sqrt{a_i \frac{x_j}{c_i} v} - x_j & \text{for } x_j \in \left(x_j^\alpha, \hat{x}_j\right) \\ 0 & \text{for } x_j \geq \hat{x}_j \end{cases} \tag{3.6}$$

with $x_j^\beta = \frac{b_i v}{4c_i}$, $x_j^\alpha = \frac{a_i v}{4c_i}$ *and* $\hat{x}_j = \frac{a_i v}{c_i}$.

Proof. See proof of Lemma 1 in Hoffmann and Kolmar (2017). □

Two of the three parts of these best response functions (abstracting from zero for $x_j \geq \hat{x}_j$) correspond to α_i and β_i respectively. We use the notion of virtual types here. We say that for AI, i.e., when player i's payoff is higher than player j's payoff, player i receives the utility of her β-type. The part of the best response function that maximises this utility is $\sqrt{b_i \frac{x_j}{c_i} v} - x_j$. For DI, when player i's payoff is lower than that of player j, she receives the utility of her α-type. This utility is maximised along $\sqrt{a_i \frac{x_j}{c_i} v} - x_j$.

The equilibria for this model are described in the following way.

Proposition 3.1 (Hoffmann and Kolmar (2017)).

There exists a unique and interior Nash equilibrium if and only if one of the cross-player inequality aversion ratios $\left(\psi_j^i = \frac{b_i}{a_j}\right)$ *is at least as high as the corresponding marginal cost ratio. More precisely,*

$$\begin{aligned} (\psi_i^j)^{-1} \geq \psi_j^i \geq \frac{c_i}{c_j} &\Leftrightarrow x_i^* = \frac{a_j b_i^2 c_j v}{(a_j c_i + b_i c_j)^2}, x_j^* = \frac{a_j^2 b_i c_i v}{(a_j c_i + b_i c_j)^2} \\ \psi_j^i < \frac{c_i}{c_j} < \left(\psi_i^j\right)^{-1} &\Leftrightarrow x_i^* = x_j^* \in [\underline{x}^*, \overline{x}^*] \end{aligned}$$

with $\underline{x}^*_{AR} = \max\left\{x_i^\beta, x_j^\beta\right\}$ *and* $\overline{x}^*_{AR} = \min\left\{x_i^\alpha, x_j^\alpha\right\}$.

Proof. See proof of Proposition 1 in Hoffmann and Kolmar (2017). □

When cost asymmetry between the players is relatively strong, each player has a strong perception of whether to expect to experience AI or DI. Thus, they act upon this inequality. Since the asymmetry is sufficient, the resulting equilibrium supports these expectations. If asymmetry is low, players enact the 'equal split' norm. If they act upon their concern for AI, the resulting payoffs would lead to DI and vice versa. Thus, there is no incentive for deviation along the equal split. Which final payoff players realise is not clear though, since between $\underline{x}^N$ and $\overline{x}^N$, every effort pair $x_i = x_j$ constitutes an equilibrium. Each player receives half of the prize in any such equilibrium. The lower the effort level along this line, the less resources are wasted.

The above proposition relies to large extents on the equivalence of $\Delta u_i = 0$ and $x_i = x_j$ since only revenues are considered. As we will see in the next section, including costs complicates this analysis.

3.2.2 Model AC: Ex-Ante & Net-Payoffs

In this model the utility function of player $i \in \{1, 2\}$ and $j \neq i$ is given by

$$U_i^{AC}(x_i, x_j) = w_i - \alpha_i \max\{-\Delta w_i, 0\} - \beta_i \max\{\Delta w_i, 0\} \tag{3.7}$$

We define the important terms $B_i = \frac{1-2\beta_i}{1-\beta_i}$ and $A_i = \frac{1+2\alpha_i}{1+\alpha_i}$. For low effort levels, difference in total costs are low. There, the main driver for the net payoff comparison is the winning probability. When cost differences are multiplied with higher efforts, the CSF becomes less sensitive to increased effort and the total level of costs has a higher influence on the net payoff comparison. Thus, the α-parts and β-parts of player 1's best response functions can have disjoint regions of x_2 in which they are a part of the actual best response function of player 1.

Lemma 3.2.
The best response function for player i is given by

$$BR_i^{AC}(x_j) = \begin{cases} \sqrt{B_i \frac{x_j}{c_i} v} - x_j & \text{if } x_j \in [0, x_j^\beta] \cup [\hat{x}_j^\beta, \frac{B_i}{c_i} v] \\ \min\left\{\pm \frac{v-(c_i-c_j)x_j-\sigma'}{2c_i}\right\} & \text{if } x_j \in (x_j^\beta, x_j^\alpha) \cup (\hat{x}_j^\alpha, \min\{\hat{x}_j^\beta, v\}) \\ \sqrt{A_i \frac{x_j}{c_i} v} - x_j & \text{if } x_j \in [x_j^\alpha, \min\{\hat{x}_j^\alpha, \frac{A_i}{c_i} v\}] \\ 0 & \text{else} \end{cases} \tag{3.8}$$

with

$$\sigma' = \sqrt{v^2 - 2v(3c_i - c_j)x_j + (c_i + c_j)^2 x_j^2}$$

$$x_j^\alpha = \frac{\left((A_i+2) - \sqrt{(A_i^2+4) - 4\frac{A_i}{c_i}}\right)^2}{4A_i(c_i+c_j)^2} c_i v \quad \hat{x}_j^\alpha = \frac{\left((A_i+2) + \sqrt{(A_i^2+4) - 4\frac{A_i}{c_i}}\right)^2}{4A_i(c_i+c_j)^2} c_i v$$

$$x_j^\beta = \frac{\left((B_i+2) - \sqrt{(B_i^2+4) - 4\frac{B_i}{c_i}}\right)^2}{4B_i(c_i+c_j)^2} c_i v \quad \hat{x}_j^\beta = \frac{\left((B_i+2) + \sqrt{(B_i^2+4) - 4\frac{B_i}{c_i}}\right)^2}{4B_i(c_i+c_j)^2} c_i v.$$

Before we discuss the different possible graphs for the best response functions, we want to introduce the general structure of the diagrams. Consider Figure 3.2.2 . Along the solid line, both players receive equal payoffs ($\Delta w_1 = \Delta w_2 = 0$). We refer to this as the *equal net* *payoff line*. It is below the 45°-line because player 2 has higher costs than player 1 ($c_2 > 1$). In the grey shaded area, below the equal net payoff line, player 1 has a lower net payoff

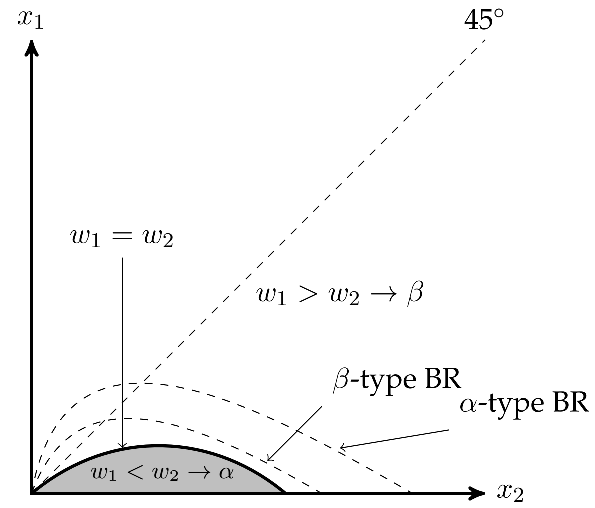


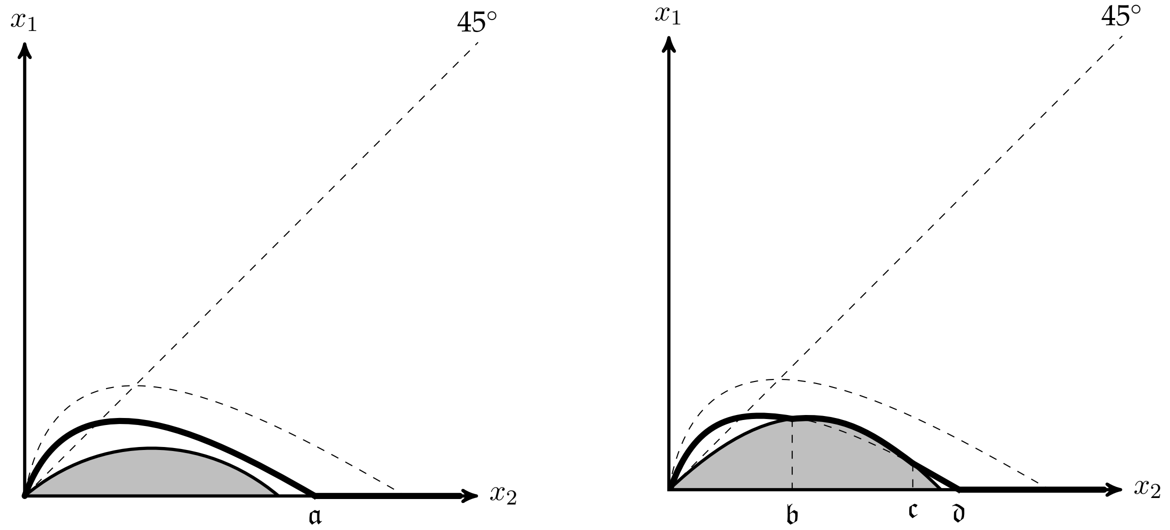
Figure 3.2.2 : Coordinate System

Note: The α-type BR refers to the part of $BR_1^{AC}(x_2)$ that is described by $\sqrt{A_1 x_2 v} - x_2$, the β-type BR refers to the part described by $\sqrt{B_1 x_2 v} - x_2$.

than player 2 and thus player 1 receives the utility of her α-type. Above the equal net payoff line, in the white area, her net payoff is higher than that of player 2. There, her best response function maximises the utility of her β-type. We use this pattern of shading of the graphs throughout the subsequent discussion. The dashed lines (beside the 45°-line) are the best response functions under the two types' utilities. The upper curve is the α-type's best response and the lower curve is the β-type's best response. Our assumptions on α and β ensure that the α-type's best response always lies above the β-type's best response. We use these two functions, together with the regions where player 1 receives the respective type's utility, to determine her actual best response graph. We are now ready to discuss the different cases of player 1's complete best response functions.

Figure 3.2.3 (a) depicts a case with very high cost asymmetry. In this case, every best response of the β-type is consistent with player 1 receiving a higher payoff than player 2 (i.e. we are still in the β-region). For any x_2 greater than $\mathfrak{a}$, player 2's effort is so high that any positive effort from player 1 would result in $U_1^{AC}(x_1, x_2) < 0$. This violation of the 'participation constraint' renders an effort 0 her best reply for $x_2 > \mathfrak{a}$.

In panel (b), cost asymmetry is still high, but in the range from $\mathfrak{b}$ to $\mathfrak{c}$ the β-type's best response would take us into the (shaded) α-region where player 1's net payoff is lower than that of player 2. This would be a contradiction, because then player 1 would not receive her β-type's utility but her α-type's utility. Conversely, the α-type's best response would be in the β-region – again a contradiction. Therefore it must be true that player 1 chooses the effort that equalises payoffs (along the thin solid line). When x_2 becomes larger than $\mathfrak{c}$,



(a) $BR_1^{AC}(x_2)$ with Very High Cost-Asymmetry

(b) $BR_1^{AC}(x_2)$ with High Cost-Asymmetry

Figure 3.2.3 : Player 1's Best Response Function for High Cost Asymmetry
Note: The thick solid line represents player 1's best response ($BR_1^{AC}(x_2)$). The light grey area represents the area where player 1 receives the β-type's utility and the white area represents the area where she receives the α-types utility. Along the solid line, player 1's and player 2's net payoffs are equal.

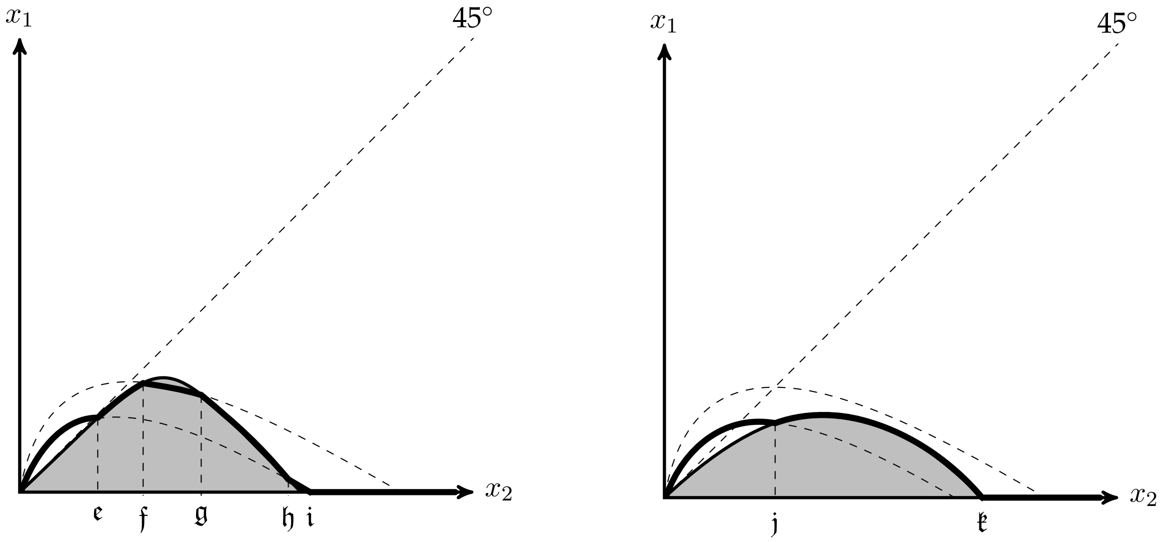
player 2's cost increase further and there is another range ($\mathfrak{c}$ to $\mathfrak{d}$) in which the effort that maximises the β-type's utility is indeed in the β-type's utility region (white area). If $x_2 > \mathfrak{d}$, player 1's 'participation constraint' $U_1^{AC}(x_1, x_2) \geq 0$ is violated for every $x_1 > 0$ (and thus $BR_1^{AC}(x_2) = 0$).

In Figure 3.2.4 we see two possible cases for medium cost asymmetry. Since the costs of player 2 are now lower than in the previous graph, the curve is less strongly bent away from the 45°-line. Panel (a) shows that for low x_2, player 1 is maximising her β-type's utility. In this range (0 to $\mathfrak{e}$) she receives a higher payoff than player 2. For the range between $\mathfrak{e}$ and $\mathfrak{f}$, she responds with efforts that result in equal payoff. The reasoning is as before: in that range of efforts each type's best response would lead into the region of the other type. Thus, there is no profitable deviation from exerting efforts on the thin solid line where net payoffs are equal. For efforts between $\mathfrak{f}$ and $\mathfrak{g}$, the effort levels that maximise player 1's α-type utility are consistent with that type (i.e. they result in higher net payoffs for player 2 than for player 1). Again, there is another range (between $\mathfrak{h}$ and $\mathfrak{i}$) in which the β-type can be supported. Eventually for $x_2 > \mathfrak{i}$, player 2's efforts become prohibitive for player 1 ($BR_1^{AC}(x_2) = 0$).

In panel (b), the equal payoff line lies strictly between the two types's best response functions for any $x_2 > \mathfrak{j}$. Thus, between $\mathfrak{j}$ and $\mathfrak{k}$, the corresponding segment of the equal payoff line is part of the best response function of player 1, as per the earlier argument. For $x_2 > \mathfrak{k}$, player 1 exerts no effort since for any positive effort her utility would be negative.[5]

Figure 3.2.5 completes the picture for low cost asymmetries. Panel (a) shows that the β-

[5] In fact we have $U_2^{AC}(\mathfrak{k}) = U_1^{AC}(0) = 0$.



(a) Medium Cost-Asymmetry 1

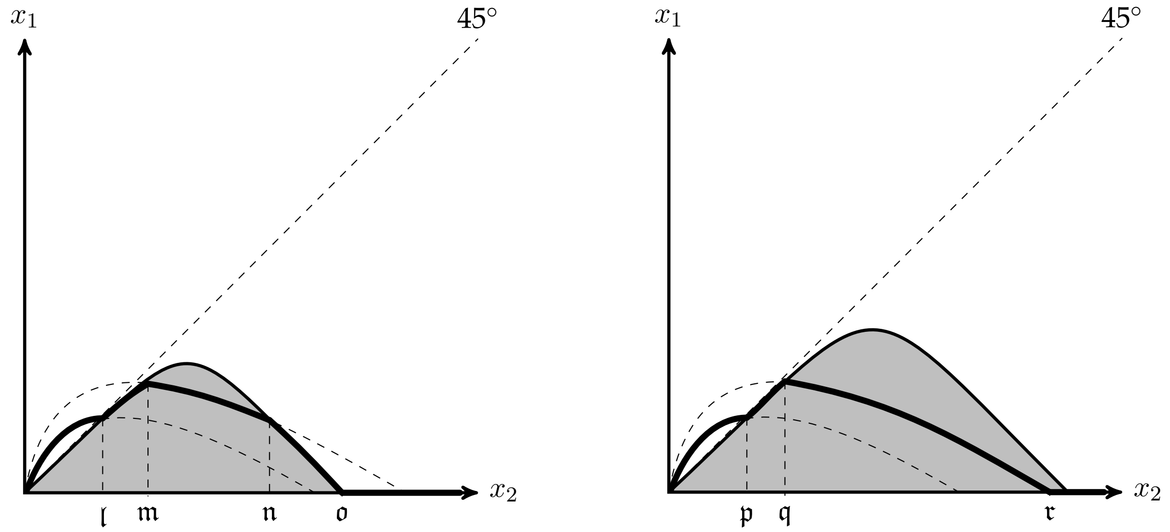
(b) Medium Cost-Asymmetry 2

Figure 3.2.4 : Player 1's Best Response Function for Medium Cost Asymmetry
Thick solid line: player 1's best response ($BR_1^{AC}(x_2)$), light grey area: player 1 receives β-type utility, white area: player 1 receives α-type utility, thin solid line: player 1's and player 2's net payoffs are equal.

type's optimal effort for $x_2 < \mathfrak{l}$ is consistent with player 1's utility being higher than that of player 2. Between $\mathfrak{l}$ and $\mathfrak{m}$ there is no profitable deviation from the equal payoff line again. Between $\mathfrak{m}$ and $\mathfrak{n}$, the α-type's optimal effort is consistent with $w_1 < w_2$. In the range from $\mathfrak{n}$ to $\mathfrak{o}$, the equal payoff line lies strictly between the two types' best response function again, thus there is no profitable deviation. For $x_2 > \mathfrak{o}$, player 1 exerts no effort.

Finally, in panel (b) the β-type is relevant for $0 < x_2 < \mathfrak{p}$ and player 1 exerts efforts that lead to equal net payoffs if $x_2 \in (\mathfrak{p}, \mathfrak{q})$. The optimal effort according to the α-type is in this case consistent with $w_1 < w_2$ for the entire range $\mathfrak{q}$ to $\mathfrak{r}$, above which player 1's participation constraint binds and she plays $BR_1^{AC}(x_2) = 0$. In summary we see that the best response dynamics for the favourite are more complex across different levels of cost asymmetry, once these costs are accounted for in the payoff comparison. With an opponent of equal strength a player only needs to consider whether he is expecting to win or lose. The incorporation of costs allows for multiple cases in which payoffs are expected to differ either way. This can either be predominantly through discrepancies in expected revenue for low effort ranges, where the CSF reacts strongly to changes in efforts, or through differences in effort costs for higher effort ranges, where the marginal probabilities hardly change, but costs are still linearly increasing.

In order to discuss player 2's best response function, we display x_2 on the y-axis and x_1 on the x-axis. For consistency, in the following graphs, the grey shaded area still represents the effort combination for which player 1's net payoff is lower than player 2's net payoff. It represents player 1's α-region and thus, in these graphs, player 2's β-region.



(a) Low Cost-Asymmetry

(b) Very Low Cost-Asymmetry

Figure 3.2.5 : Player 1's Best Response Function for Low Cost Asymmetry
Thick solid line: player 1's best response ($BR_1^{AC}(x_2)$), light grey area: player 1 receives β-type utility, white area: player 1 receives α-type utility, thin solid line: player 1's and player 2's net payoffs are equal.

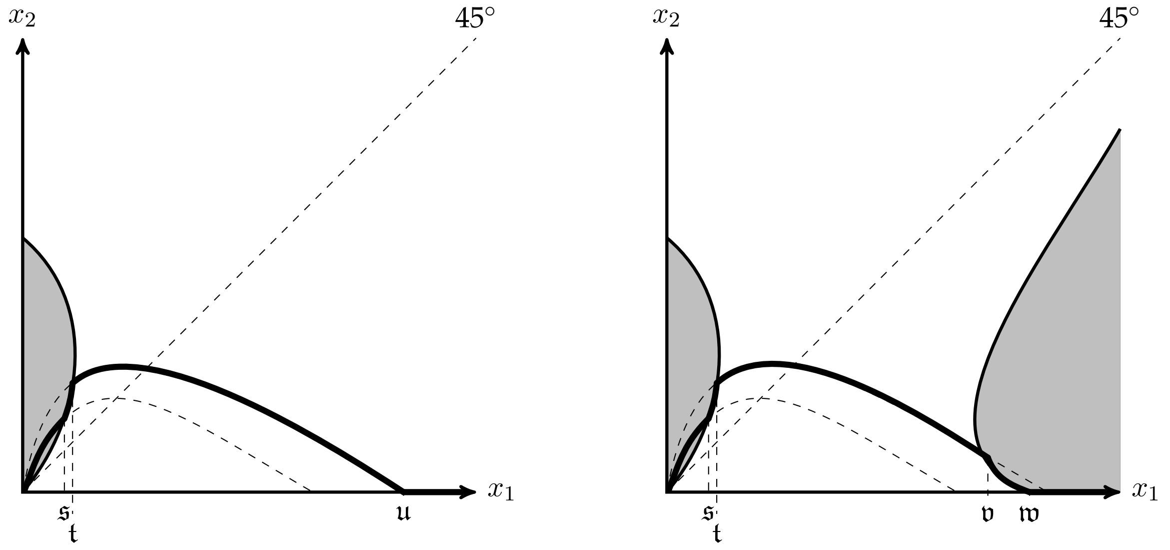
Player 2's best response function is considerably simpler to discuss and is structurally always identical to those in Hoffmann and Kolmar (2017).[6]

In panel (a) of Figure 3.2.6 , for $0 < x_1 < \mathfrak{s}$, player 2's net payoff is higher than player 1's net payoff. Consequently, maximising player 2's β-utility is optimal. In the range between $\mathfrak{s}$ and $\mathfrak{t}$, there is no profitable deviation for player 2 from exerting the effort that equalises net payoffs. Higher efforts are in the region where she receives the β-type's utility, which is maximised at an effort level below the equal net payoff line. Lower efforts lie in the region where the α-type's best response is maximising her utility, but these effort lie in the β-region.

For efforts $x_1 > \mathfrak{t}$, all of the remaining part of the α-type result in maximal utility for player 2, while being in the white region with $w_1 > w_2$. The participation constraint binds for $x_1 > \mathfrak{u}$ and thus player 2 exerts zero effort beyond this point.

The left half of panel (b) is identical to panel (a). The right half points to an issue where player 1 exerts enough effort to (almost) fully dissipate the prize. In the most extreme case, with $x_1 = v$ and $x_2 = 0$, the favourite receives the prize for sure and has costs equal to v. Thus, her net payoff is $p(v, 0)v - v = 1v - v = 0$. Player 2 loses for sure but also does not exert any effort resulting in zero costs. Her net payoff is $(1 - p(v, 0))v - 0 = 0v - 0 = 0$ as well. Varying the efforts from this point in the right way, this produces a second equal net payoff line. It lies in the higher range of x_1 in Figures 3.2.3 to 3.2.5 . We omitted it there though, as it is strategically irrelevant for the equilibrium analysis. To see this consider the following. The lowest level of x_1 on this second equal net payoff line is $\frac{1}{2}v$ when $c_2 = 1$.

[6] In the sense that each type's utility function is only relevant in one range of the opponent's effort. These ranges again are separated by effort combinations where there are no deviation incentives to efforts on the equal payoff line.



(a) Higher Cost Asymmetry

(b) Lower Cost-Asymmetry

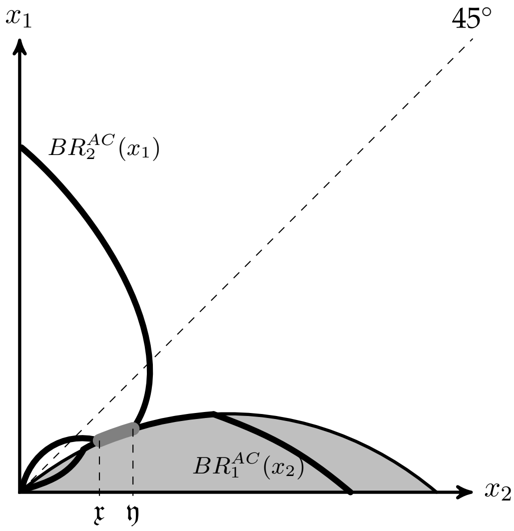
Figure 3.2.6 : Best Response Functions for Player 2

Note: The thick solid line shows player 2's best response function. The grey areas represent the regions where player 2 receives the β-type's utility. The white area represents the region where she receives the α-type's utility. Along the thin solid lines both players receive the same payoff.

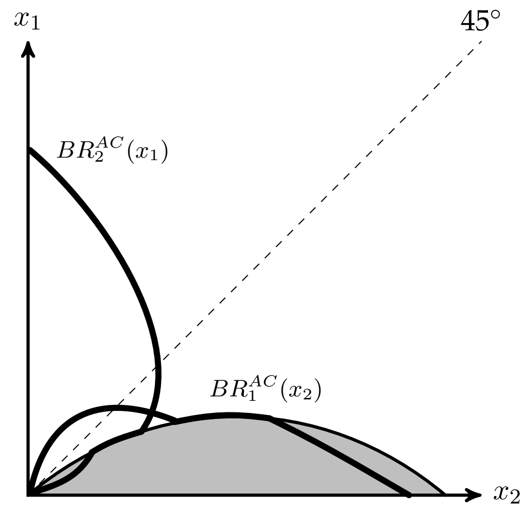
But the highest possible level of player 1's best response is the maximum of her α-type's best response, which is $\frac{A_1}{4}v < \frac{1}{2}v$. Thus, it always lies below the smallest possible x_1 on the second equal net payoff line. An intersection with the β-type's best response function is only possible for an inequality prone player, but the same argument as for the α-type ensures that this is irrelevant in equilibrium.

Consider the possible types of equilibria in this model, shown in Figure 3.2.7 . Qualitatively, the equilibrium dynamics are similar to the ones in Hoffmann and Kolmar (2017). In panel (a), the two best response functions overlap on their segments that lie on the equal net payoff line. The equilibria are thus given by all efforts of player 2 in the range from $\mathfrak{x}$ to $\mathfrak{y}$ and the corresponding best responses of player 1. In panel (b), the segment of player 1's best response function that corresponds to her β-type intersects with the segment of player 2's best response function that corresponds to her α-type. Panel (c) shows the reverse case.

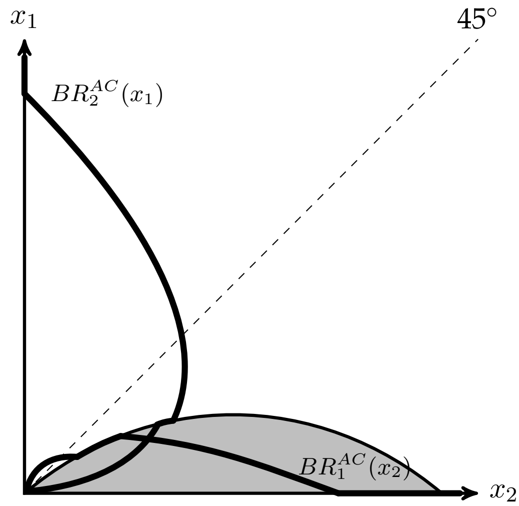
While in Hoffmann and Kolmar (2017), for $c_2 < \underline{c}$, the favourite can never end up with less than half the prize, this is indeed possible when costs are considered. This is illustrated by panel (b) of Figure 3.2.7 . Thus, if the favourite is inequality averse, she willingly forgoes a part of her prize to 'make up' for the high costs her opponent is facing. For this to happen, the underdog needs to be sufficiently DI averse in order for this equilibrium to lie below the $45°$-line. The case for an equilibrium where player 1's α-type and player 2's β-type are active is more extreme in this model, as for stronger forms of cost asymmetry, the equal payoff curve is below that intersection and thus the two types are not relevant. The formal description of these equilibria is given in the following proposition.



(a) Continuum of Equilibria



(b) Player 1 Maximises β-Type Utility, Player 2 Maximises α-Type Utility



(c) Player 1 Maximises α-Type Utility, Player 2 Maximises β-Type Utility

Figure 3.2.7 : Possible Equilibria in Model AC

Proposition 3.2.

Let $\underline{c} = \frac{B_2}{A_1}\left(\frac{1}{2}(B_2 - A_1) + \sqrt{\frac{1}{4}(B_2 - A_1)^2 + 1}\right)$ *and* $\overline{c} = \frac{A_2}{B_1}\left(\sqrt{\frac{1}{4}(A_2 - B_1)^2 + 1} - \frac{1}{2}(A_2 - B_1)\right)$.

If $c_2 < \underline{c}$ *with* $\underline{c} > 1$, *there exists a unique and interior Nash equilibrium given by*

$$(x_1^*, x_2^*) = \left(\frac{c_2 A_1^2 B_2}{(c_2 A_1 + B_2)^2} v, \frac{A_1 B_2^2}{(c_2 A_1 + B_2)^2} v\right). \tag{3.9}$$

If $c_2 > \overline{c}$ *with* $\overline{c} > 1$, *there exists a unique and interior Nash equilibrium given by*

$$(x_1^*, x_2^*) = \left(\frac{c_2 A_2 B_1^2}{(A_2 + c_2 B_1)^2} v, \frac{A_2^2 B_1}{(A_2 + c_2 B_1)^2} v\right). \tag{3.10}$$

This is only possible if $-\beta_2 > \alpha_1$ *and thus* $\beta_2 < 0$ *and* $B_2 > A_1$.

If $c_2 \in (\underline{c}, \overline{c})$, *there exists a continuum of Nash equilibria given by*

$$(x_1^*, x_2^*) = \left\{(x_1 = BR_1^{AC}(x_2), x_2) | x_2 \in [\max(x_2^\beta, BR_2^{AC}(x_1^\beta)), \min(x_2^\alpha, BR_2^{AC}(x_1^\alpha))]\right\} \tag{3.11}$$

As we have seen from the discussion of the best response graphs, the best response functions can fail to have a segment on the equal payoff line in this model (see, e.g., Figure 3.2.3 (a)). We can give a sufficient condition for when this is not the case for both players. This guarantees that the set of possible choices for $c_2 > 1$ that result in a continuum of equilibria is not empty.

Lemma 3.3.
The set $(\underline{c}, \overline{c})$ is nonempty if

$$\left(1 - \frac{A_2}{B_1}\right) A_2 + \left(1 - \frac{B_2}{A_1}\right) B_2 \geq 0 \tag{3.12}$$

The model is algebraically more demanding and thus does not allow to pin down the exact closed form for the respective combinations of cost and inequality aversion ratios. It should be apparent from Figure 3.2.7 (a) though, that through the curvature of the equality-curve, the range and level of total efforts for the stronger player is reduced relative to Model AR. We will see that this implies a difference in the effect that cost asymmetry has on total efforts. Just as in the previous model, the continuity of equilibria can account for overspreading of bids, but the compression of bids for player 1 also implies a different extent of this prediction.

3.2.3 Digression: A Robustness Check for Multiplicity

Applying the model of Fehr and Schmidt (1999) with ex-ante comparisons to contest leads to multiplicity of uncountably many equilibria. This is irrespective of wether agents incorporate each other's costs or not. The key underlying assumption though is that they agree on what is fair and that they both either consider the costs or not. The only heterogeneity we allow for is how much they suffer under unfairness relative to the 50-50 norm of either revenues or net-payoffs. There are two ways (and their combinations) that make these continua of equilibria disappear.
First, consider an alternative way to model social preferences could be

$$U_i^A = w_i - 2\left(\alpha_i \max\{\lambda_i u_j - (1-\lambda_i)u_i, 0\} - \beta_i \max\{(1-\lambda_i)u_i - \lambda_i u_j, 0\}\right) \tag{3.13}$$

with $\lambda_i \in (0,1)$.[7] It represents how much player i thinks she should obtain as a share of the prize. If, say, player 1 has the 50-50 norm ($\lambda_1 = 0.5$), but player 2 thinks she should receive

[7] We chose a comparison on revenues for this example rather than net-payoffs, but the argument carries over to the other ex-ante setting. Also, we could have chosen $\lambda \in (0,2)$ straight away, but here the middle of the interval has the nice interpretation of a $50-50$ split.

more to compensate her higher costs ($\lambda_j > 0.5$), the resulting equilibrium is unique.[8]
For $\lambda_i = \frac{1}{2}$ for both i, we are in Model AR. Whenever $\lambda_i \neq \lambda_j$, the situation is qualitatively as illustrated in Figure 3.2.8 . Alternatively, player 1 might consider a 50-50 split of revenues as

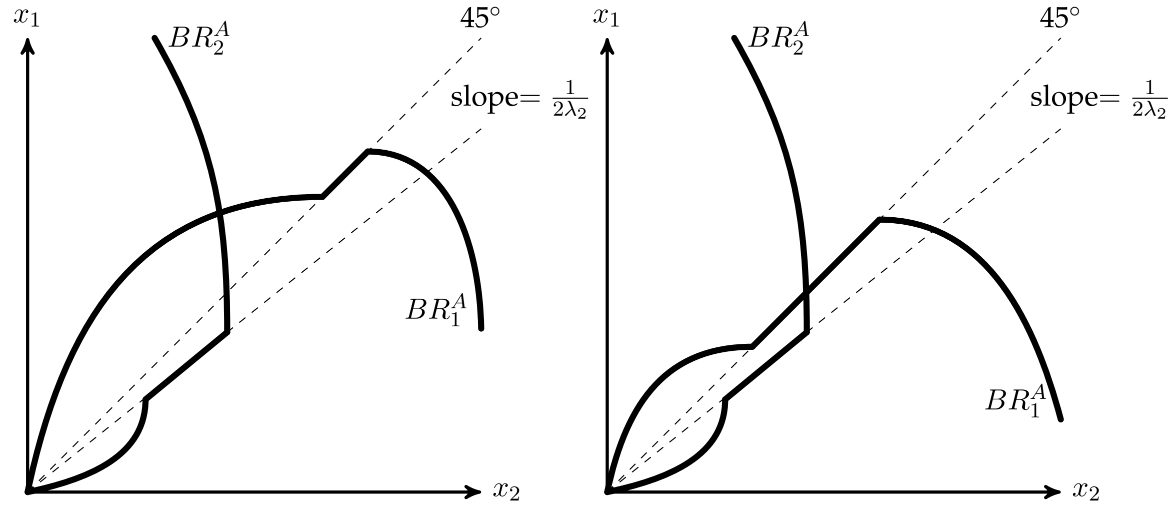


Figure 3.2.8 : Equilibria with Different Fairness Norms $\left(\lambda_2 > \frac{1}{2}\right) = \lambda_1$

fair, while player 2, who incurs higher costs, considers equal net-payoffs as fair. Again, the resulting equilibrium is unique and it fails to offer an explanation of overspreading of bids in contest. Since the aim of this paper is to discriminate between social preference models

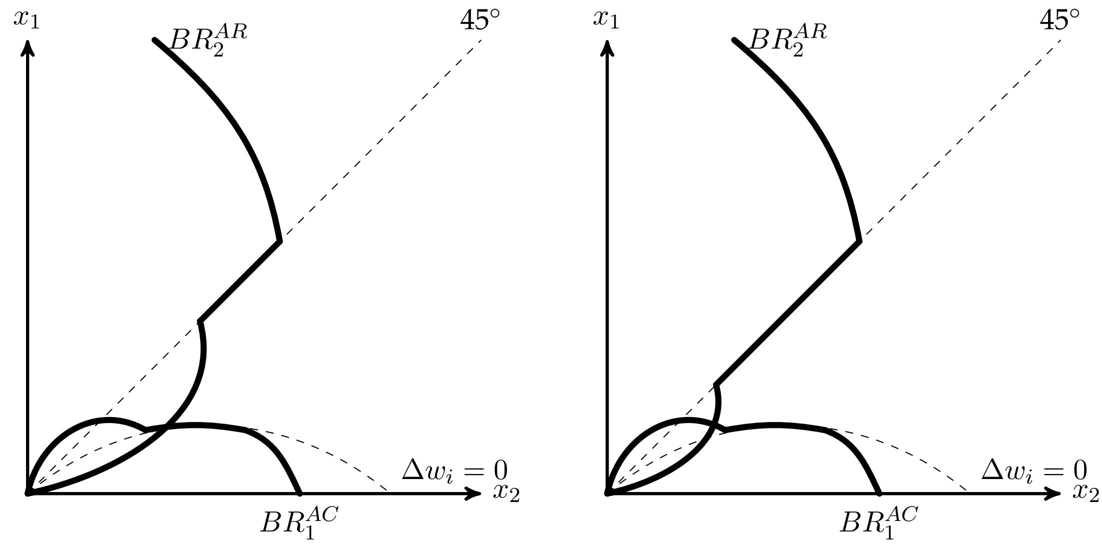


Figure 3.2.9 : Ex-ante vs. Ex-post Player

in contest in order to decide which one is preferable in theory and/or practice, this section is not meant to introduce yet another model. It illustrates that the multiplicity of equilibria, which is the main advantage of the ex-ante models in terms of describing phenomena in actual contest data, is not robust to heterogeneity in terms of fairness norms and considerations of costs. Although the 50-50 norm is relatively focal, it is not the only possible fair

[8]We omit the proof here, but the argument mainly follow from the fact that the 'fair' lines do not intersect and separate the $\mathbb{R}^2_+$-plane into two regions for the α and β type respectively. Since any intersection between any two of those curves is unique in $\mathbb{R}^2_{++}$ and $(0,0)$ cannot be an equilibrium through variational arguments, the result follows.

allocation of the prize. More importantly, whether real individuals consider costs in social comparisons or not, is not generally clear.

3.2.4 Model PC: Ex-Post & Net-Payoffs (Herrmann and Orzen, 2008)

The utility function is given by

$$\begin{aligned} U_i^{PC} &= w_i - p_j\alpha_i\Delta z_j - p_i\beta_i\Delta z_i \\ &= (1+\alpha_i-\beta_i)p_i v - \alpha_i v - c_i x_i + (p_i\beta_i - (1-p_i)\alpha_i)(c_i x_i - c_j x_j) \\ &= U_i^{PR} + (p_i\beta_i - (1-p_i)\alpha_i)(c_i x_i - c_j x_j) \end{aligned} \tag{3.14}$$

where the last line is given as a means of comparison with the next and final Model PR. The best response functions for this model can be characterised completely, also for $c_i \neq c_j$, which is not covered in Herrmann and Orzen (2008).

Lemma 3.4.
The best response functions for Model PC for each $i \in \{1, 2\}$ are given by

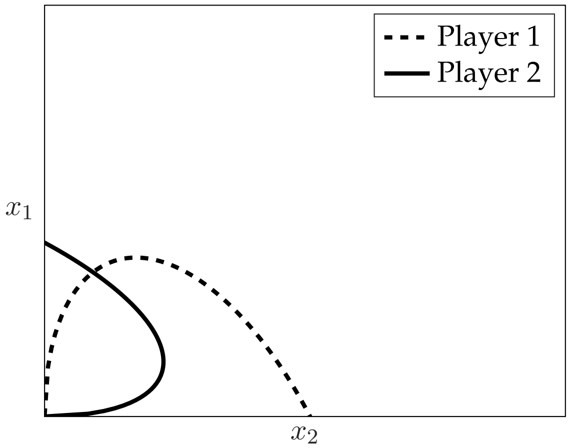
$$BR_i^{PC}(x_j) = \begin{cases} \frac{\sqrt{(1+\alpha_i-\beta_i)v-(\alpha_i+\beta_i)(c_i+c_j)x_j}}{\sqrt{1-\beta_i}}\sqrt{\frac{x_j}{c_i}} - x_j & \text{if } x_j < \overline{\overline{x}} \\ 0 & \text{if } x_j \geq \overline{\overline{x}} \end{cases} \tag{3.15}$$

with

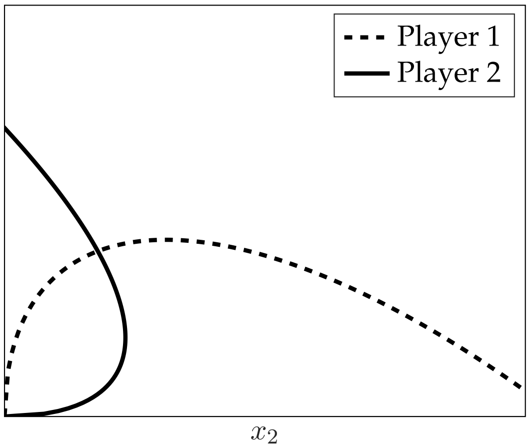
$$\overline{\overline{x}} = \frac{(1+\alpha_i-\beta_i)v}{c_i+\beta_i c_j+\alpha_i(c_i+c_j)}.$$

The solution of this system of best response functions is technically intractable for any other case than $c_i = c_j$, $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$. We provide two graphs from numerical investigations to illustrate possible equilibria in this model.[9] Herrmann and Orzen (2008) suggest that, for a simple modification of this expression, it is possible to estimate the parameters from observed efforts and parameters of an experiment. In their procedure, they first use the selfish model to estimate the 'virtual valuations' V of each player. It measures how much subjects value the prize, which is possible different from the monetary incentive.[10] This is done to net out 'joy of winning' as an alternative explanation for overbidding. The best response functions in that case are $x_i = \sqrt{\frac{V}{c_i}x_j} - x_j$, which can be rewritten as $x_i + x_j = \sqrt{\frac{V}{c_i}x_j}$. Using the strategy method, asking each subject to state their desired

[9]The equilibrium for the symmetric case mentioned here is given in Herrmann and Orzen (2008) as $\frac{1+\alpha-\beta}{2+\alpha-\beta}\frac{v}{2}$.
[10]For a prize of 16 tokens, their estimate for the two waves of subjects are 23.05 and 24.40.



(a) $\alpha = 0.6$, $\beta = 0.3$



(b) $\alpha = 0.7$, $\beta = -0.3$

Figure 3.2.10 : Potential Equilibria in Model PC

Note: Marginal costs of player 2 are set to $c_2 = 2$.

response r_s to each strategy $s \in \{1, 2, ..., S\}$, they estimate

$$t_s = \theta_1 o_s + \epsilon_s \tag{3.16}$$

with $t_s = \overline{r}_s + s$, $o_s = \sqrt{s}$ and $\theta_1 = \sqrt{\frac{V}{c_i}}$, where $\overline{r}_s$ is the mean response of all subjects in the respective wave.[11] They then use the resulting value V in a regression after taking the best response functions in (3.15) and transforming them to

$$(x_i + x_j)^2 = \frac{(1 + \alpha_i - \beta_i)V}{1 - \beta_i} \frac{x_j}{c_i} - \frac{(\alpha_i + \beta_i)(c_i + c_j)}{1 - \beta_i} \frac{x_j^2}{c_i}. \tag{3.17}$$

The final regression then is given by

$$t_s^2 = \theta_2 o_s + \theta_3 o_s^2 + \eta_s \tag{3.18}$$

with $\theta_2 = \frac{(1 + \alpha_i - \beta_i)V}{(1 - \beta_i)c_i}$ and $\theta_3 = \frac{(\alpha_i + \beta_i)(c_i + c_j)}{(1 - \beta_i)c_i}$. From the estimates of this equation, one can back out the social preference parameters.[12]

Herrmann and Orzen (2008) use symmetric costs in there experiments. The analysis of asymmetric costs reveals a problem with this procedure. It should be apparent from these expressions that the estimated social preference parameters depend on the level of costs and the cost asymmetry. For example, for the values of $\hat{\theta}_2$ and $\hat{\theta}_3$ provided by Hoffmann and

[11] In their experiment, subjects had to choose a best reply to all integer efforts from 1 to 16.

[12] In Herrmann and Orzen (2008) those are $\hat{\alpha}_{one} = 1.551$ and $\hat{\beta}_{one} = -1.403$ for the first part of their experiment and $\hat{\alpha}_{two} = 0.589$ and $\hat{\beta}_{two} = -0.287$ for their second part.

Kolmar (2017), the following comparative statics obtain at $c_1 = c_2 = 1$.

$$\frac{\partial \hat{\alpha}_i}{\partial c_i} < 0, \quad \frac{\partial \hat{\alpha}_i}{\partial c_j} < 0$$

and

$$\frac{\partial \hat{\beta}_i}{\partial c_i} < 0, \quad \frac{\partial \hat{\beta}_i}{\partial c_j} > 0$$

These seem to suggest the following: If own costs increase and $\beta_i > 0$, the player becomes less sensitive to AI, as her concern for her own costs increases. If $\beta_i < 0$ she becomes even more spiteful the higher her own costs. An individual with higher costs is more likely to be spiteful in the first place. For α_i, an increase in c_j reduces player i's concern for DI. The same is true for an increase in c_i.

The problem with this approach is, that it starts out with the assumption of exogenous preferences. This is in contradiction with the estimated parameters, as they suggest social preferences are endogenous and depend on the level of costs. This problem is hidden for equal costs, but that does not mean that the estimated parameters are exogenous of costs. Model PC thus only seemingly has an advantage over the ex-ante formulations in terms of being implementable in OLS. A new technique, that is beyond the scope of the paper at hand, should implement a restriction that social preference parameters are independent from cost structures.

We will see that the same conclusion arises in the next and final model.

3.2.5 Model PR: Ex-Post & Revenues

In this model, agents base their comparison on ex-post revenues. The problem, relative to that of a selfish player, is scaled up or down, depending on parameters. We define $d_i = 1 + \alpha_i - \beta_i$. The utility function is given by

$$\begin{aligned} U_i^{PR} &= w_i - p_j \alpha_i (\Delta y_j) - p_i \beta_i \Delta y_i \\ &= d_i p_i v - \alpha_i v - c_i x_i \end{aligned} \tag{3.19}$$

Since the non-linearities of the CSF only enter once, this problem is more tractable than the ex-ante models and we can derive smooth and unique best response functions.

Lemma 3.5.

The best response functions for Model PR for each $i \in \{1,2\}$ are given by

$$BR_i^{PR}(x_j) = \begin{cases} \sqrt{d_i \frac{x_j}{c_i} v} - x_j & \text{if } x_j < \frac{d_i}{c_i} v \\ 0 & \text{if } x_j \geq \frac{d_i}{c_i} v \end{cases} \tag{3.20}$$

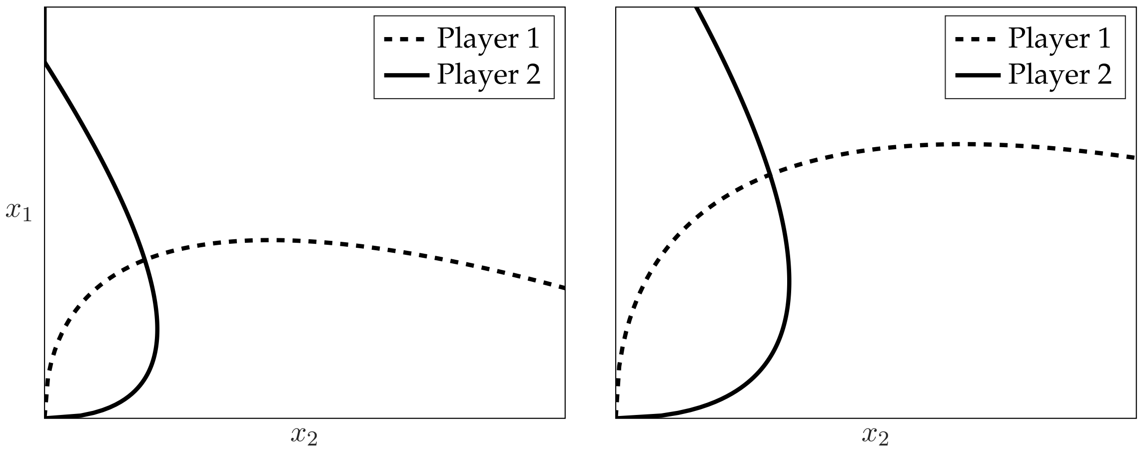
From there, we can use the fact that these functions are identical to the standard Tullock model for $\tilde{v} = \frac{d_i}{c_i} v > 0$ and conclude the uniqueness of equilibrium from the well-known results in Nti (1999) and Nti (2004). We omit the proof here.

Corollary 3.1.

The unique equilibrium is given by

$$(x_1^*, x_2^*) = \left(\frac{d_1 d_2^2 c_2}{(d_1 + d_2 c_2)^2} v, \frac{d_1^2 d_2}{(d_1 + d_2 c_2)^2} v \right) \tag{3.21}$$

The equilibrium is structurally similar to the selfish Nash equilibrium. This is due to the fact that the best response functions are scaled versions of those in the selfish case. The model can thus predict overbidding (and overdissipation) relative to the selfish Nash equilibrium for the appropriate parameters. Within the bounds of our parameters, this model is thus qualitatively similar to the previous model while being technically more tractable. Model PR leads to the same conclusion of endogenous preferences as Model PC. Additionally, the virtual valuation V and the social preferences d_i cannot easily be disentangled as



(a) $\alpha = 0.6$, $\beta = 0.3$

(b) $\alpha = 0.7$, $\beta = -0.3$

Figure 3.2.11 : Potential Equilibria in Model PR

Note: Marginal costs of player 2 are set to $c_2 = 2$.

$$\frac{V}{d_i} = \frac{v}{c_i}.$$

Under the assumption that v and V are constant across individuals in the lab, this equation again provides a problematic statement on how social preferences vary with costs: The dislike of DI or enjoyment of AI (dislike of AI) increases (decreases) in the level of costs. Thus, a subject with higher costs would feel more deserving or feel her opponent is less deserving than a subject with lower costs. An approach that implements the restriction that social preference parameters are independent from costs is needed here as well. But even if such an approach was available, d_i only allows to identify a summary index of social preferences that does not allow to disentangle the two concerns separately.

3.3 Model Comparison

Abstracting from cost considerations, it depends on whether the CSF is interpreted as a probability of winning the contested prize or a share of it. For the probabilistic assignment of prizes, the models differ in terms of their underlying assumptions on how individuals form comparisons. Interpreting the CSF as a share in the ex-ante and as a probabilistic allocation rule in the ex-post models, the difference in the allocation mechanism constitutes the difference between the models.

They also differ in terms of whether pure revenues or net payoffs are considered. This naturally results in differences in both technical tractability[13] and implications of the results. Table 3.2 summarises these differences for the four models presented in this article.

	Model AR	Model AC	Model PR	Model PC
Expectations	Ex-Ante	Ex-Ante	Ex-Post	Ex-Post
Comparison	Revenues	Net Payoffs	Revenues	Net Payoffs
Existence	✓	✓	✓	✓
Tractable	✓	✓	✓	✗
Multiplicity	✓	✓	✗	✗[14]
Overbidding (Overdissipation)	✓ (✓)	✓ (✓)	✓ (✓)	✓ (✓)
Overspreading	✓	✓	✗	✗
Share vs. Prob. Contest	Share (& Prob.)	Share (& Prob.)	Prob.	Prob.
Expected Utility Formulation	Unconventional	Unconventional	Conventional	Conventional

Table 3.2: Model Comparison

All models allow for explaining overbidding and overdissipation, given the appropriate parameters.[15] Overspreading is relatively hard to explain within the ex-post models as they result in unique equilibria. Without relying on factors outside the model, the ex-ante models suggest that multiplicity of equilibria and failure to coordinate on the most efficient (or any

[13]The term tractable here refers to whether equilibria can be characterised in closed form.

[14]Multiplicity can occur for values of α and β that we have excluded in this analysis. Multiplicity occurs in three possible equilibria that are qualitatively similar to the ones described in Chowdhury and Sheremeta (2011).

[15]Overbidding occurs for the standard parameters found in the laboratory in other studies. Underbidding is possible but needs higher βs and αs than typically found.

other) equilibrium constitute a lasting spread around or above the Nash equilibrium in the selfish model. Resorting to other explanations for this variation, like best responses played to previous rounds as in Wärneryd (2018) (particularly in the presence of cost asymmetries) and understanding difficulties of the implemented CSF as in Chowdhury et al. (2017) dramatically reduces the advantage of these types of models. Furthermore, and as mentioned above, the ex-ante models are more applicable to the share interpretation of the CSF. Chowdhury et al. (2014) demonstrate that subjects in this type of contests actually seem to exhibit considerably less overspreading than under the probabilistic allocation rule.

3.3.1 Level of Total Efforts Across Models and the Degree of Cost Asymmetry

The total effort in equilibrium, given the model $m \in \{AR, AC, PR, PC\}$, is given by

$$X_m^* = x_{1,m}^* + x_{2,m}^* \tag{3.22}$$

In the case of Model AR and AC and $c_2 < \min\{\psi_1^2, (\psi_2^1)^{-1}, \underline{c}\}$ we have

$$X_{AR}^* = \frac{a_1 b_2}{(a_1 c_2 + b_2)} v \text{ and } X_{AC}^* = \frac{A_1 B_2}{(A_1 c_2 + B_2)} v$$

One can find that

$$\frac{\beta_2}{\alpha_1} \frac{1 + 2\alpha_1}{1 - 2\beta_2} < \frac{1}{c_2} \Leftrightarrow X_{AR}^* > X_{AC}^*$$

Recall that this equilibrium can only occur in both models if player 1 is inequality prone ($\beta < 0$). Thus, the left expression and consequently the comparison of X_{AR}^* and X_{AC}^* holds, whenever we are in that equilibrium. At some level of effort for player 1, her opponent is discouraged and reduces her effort. But that also reduces the costs for player 2, which player 1 incorporates in her enjoyment of AI. Thus, she has no incentive to increase her efforts further, as opposed to Model AR where this consideration is not made, since costs are not included.

Turning to the unique equilibrium in which $c_2 > \max\{\psi_2^1, (\psi_1^2)^{-1}, \bar{c}\}$ and player 1 is necessarily inequality prone we see that

$$X_{AR}^* = \frac{a_2 b_1}{a_2 + b_1 c_2} v \text{ and } X_{AC}^* = \frac{A_2 B_1}{A_2 + B_1 c_2} v.$$

Solving the inequality between these, we find

$$c_2 \gtrless \frac{\beta_1}{\alpha_2} \frac{1 + 2\alpha_2}{1 - 2\beta_1} \Leftrightarrow X_{AR}^* \gtrless X_{AC}^*.$$

For the unique equilibria in these models it is thus apparent that $\beta_1 < 0$ always results in higher total effort in Model AR as compared to Model AC. The reason is in the consideration of costs when enjoying AI. In Model AR, increases in player 1's effort monotonically increase her AI and only decrease her direct payoff through the costs she incurs. In Model AC, her effort costs decrease both her payoff and her AI (all other things equal). For inequality averse players, we see that for any given set of parameters there is some level of c_2 such that Model AR has higher total efforts than Model AC. This hints already at a more general connection between total efforts and cost asymmetry.

The selfish model predicts that total (and individual) efforts decrease in cost asymmetry.[16] Some articles have thus investigated this prediction as it is at odds with conventional wisdom where inequality is expected to increase conflict (Stewart, 2000; Nafziger and Auvinen, 2002; Østby, 2008).[17] The experimental studies on this issue manipulated the ex-ante degree of cost/impact asymmetry between players and unanimously find declining efforts with increased cost asymmetry (see, e.g., Anderson and Stafford (2003), Fonseca (2009), Anderson and Freeborn (2010) or Kimbrough et al. (2014)). From the above expressions, for all levels of total efforts, we see that this holds true under social preferences as well, although the strong player might increase her effort depending on parameters.

Corollary 3.2.
For models $m \in \{AR, AC, PR\}$ and all $i \in \{1, 2\}$ we have the following.

$$\frac{\partial x^*_{1,AR}}{\partial c_2} \gtrless 0 \text{ if } c_2 \begin{cases} \leq \frac{b_2}{a_1} \leq \frac{a_2}{b_1} \\ \geq \frac{a_2}{b_1} \geq \frac{b_2}{a_1} \end{cases}$$

$$\frac{\partial x^*_{1,AC}}{\partial c_2} \gtrless 0 \text{ if } c_2 \lesseqgtr \frac{A_2}{B_1} \text{ and } c_2 > \overline{c}$$

$$\frac{\partial x^*_{1,AC}}{\partial c_2} \gtrless 0 \text{ if } c_2 \lesseqgtr \frac{B_2}{A_1} \text{ and } c_2 < \underline{c}$$

$$\frac{\partial x^*_{1,PR}}{\partial c_2} \gtrless 0 \text{ if } c_2 \lesseqgtr \frac{d_1}{d_2}$$

$$\frac{\partial X^*_m}{\partial c_2} < 0 \text{ if } c_2 < \underline{c} \text{ or } c_2 > \overline{c}$$

Player 2's equilibrium effort decreases in c_2 in all Models $m \in \{AR, AC, PR\}$ for unique equilibria.

Thus, social preferences applied to contest can explain higher levels of total effort but they exhibit the same comparative statics with respect to cost asymmetry.[18] For individual efforts this is not necessarily the case.

Consider Figure 3.3.12 . In the selfish model any deviation from equal costs reduces indi-

[16] This seems to apply to most forms of asymmetries, like asymmetric impact in the CSF or valuations of the prize.

[17] We are aware that these examples refer to much richer environments and encompass, e.g., grievances, history dependence and ideology. Still, the view of many early economists, that it asymmetry and/or (procedural) inequality itself that generates more conflict, is strongly at odds with the robustness of the opposite result in

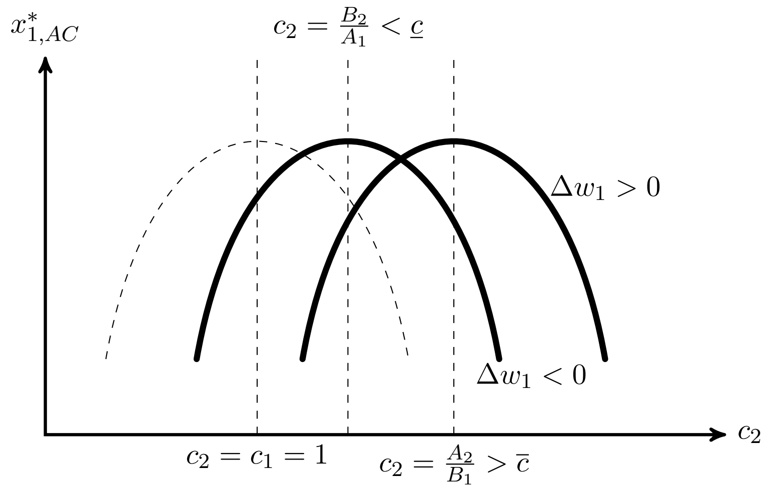


Figure 3.3.12 : Maximum of Player 1's Equilibrium Effort Shifts with Social Preferences in Model AC

Note: The dashed line illustrates how player 1's equilibrium effort changes with c_2 if she was selfish. The left curve shows her efforts over c_2 for equilibria in which she feels disadvantaged (Figure 3.2.7 , Panel (c)). The right curve represents her equilibrium effort in equilibria where she feels advantaged (Figure 3.2.7 , Panel(b)). The level of the maximum can differ across parameters, but we kept it the same as it is not the focus of the subsequent discussion.

vidual efforts. When $\Delta w_1 < 0$ in equilibrium, player 1's concern for DI increase her efforts even though her opponent has higher costs. Since this equilibrium can only occur for $\beta_2 < 0$, B_2 is large, shifting the maximum effort further to the right. If $\Delta w_1 > 0$, player 2 feels disadvantaged and thus fights harder to catch up with player 1. If player is fairly concerned about AI, i.e., B_1 is relatively low, this can shift the maximum effort considerably to the right. Referring back to Figure 3.2.7 panel (b) one can see that a downward shift of player 2's best response curve moves the intersection with $BR_1^{AC}(x_2)$ further up along the x_1-axis thus increasing effort. This can have implication for the design of contests where high or at least balanced effort is desirable, like sports competitions, procurement contests and job applications. It is particularly important there, as it is the high ability (low cost) player that might underperform. With the selfish model, the prediction is that the individual effort is highest when costs are the same. Considering social preferences this might not actually extract all the possible effort from the contestants and a slight skew in terms of prizes or impact (e.g., by changing the application procedures) might actually result in more balanced efforts.

For Model PC and the regions of the continua of equilibria expressions become intractable. This is why we partially have to resort to numerical investigations.

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theoretical contest models.

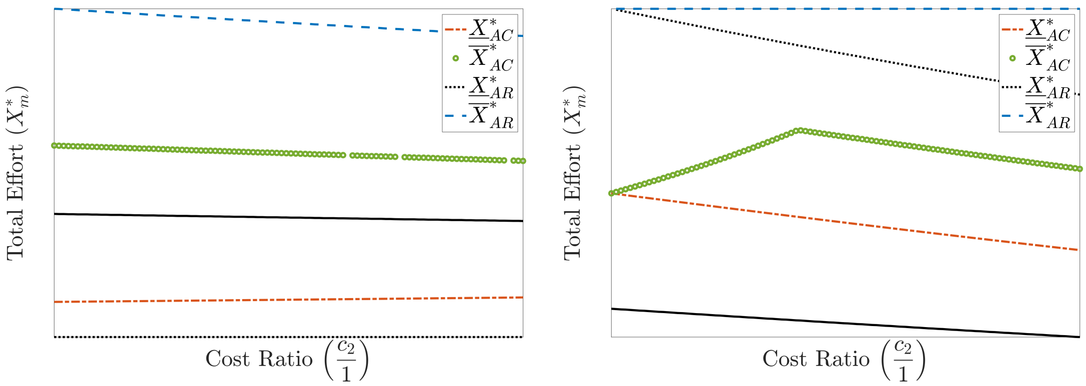
[18] As cost asymmetry is isomorphic to asymmetry in valuations and impact, this applies more widely. To see this consider $c_1 = 1$ and $c_2 = f$

$$\frac{x_2}{x_1 + x_2} v - fx = \left(\frac{x_2}{x_1 + x_2} \tilde{v} - x \right) f = \frac{\frac{\tilde{x}_2}{f}}{x_1 + \frac{\tilde{x}_2}{f}} v - \tilde{x}_2 \tag{3.23}$$

for $\tilde{v} = \frac{v}{f}$ and $\tilde{x}_2 = f x_2$.

3.3.2 Spread of Total Efforts and Degree of Cost Asymmetry

In the continua of equilibria in the ex-ante models, theoretical results are technically challenging, particular for Model AC. Using numerical explorations, we can show that, to a certain degree, the above results also seem to hold there as well. Let $\underline{x}^*_{AC} = \max x_2^{\beta}, BR_2^{AC}(x_1^{\beta})$ and $\overline{x}^*_{AC} = \min x_2^{\alpha}, BR_2^{AC}(x_1^{\alpha})$. Figure 3.3.13 shows how total effort-ranges change with increasing cost asymmetry across the two ex-ante models. In both panels we see that Model



(a) Inequality Averse Players

(b) One Inequality Prone Player

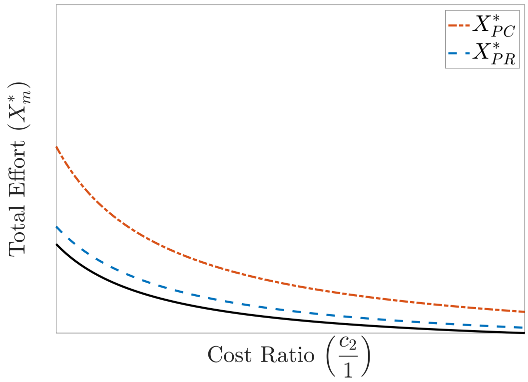
Figure 3.3.13 : Total Effort for Increasing Cost Asymmetry in the Continua of Equilibria in Models AR & AC

Note: The graph plots $\underline{X}^*_{AC} = \underline{x}^*_{AC} + BR_1^{AC}(\underline{x}^*_{AC})$ and $\overline{X}^*_{AC} = \overline{x}^*_{AC} + BR_1^{AC}(\overline{x}^*_{AC})$ versus $\underline{X}^*_{AR} = 2\underline{x}^*_{AR}$ and $\overline{X}^*_{AR} = 2\overline{x}^*_{AR}$. As a benchmark, the black solid line represents the Nash equilibrium in case of two selfish players. The left graph is obtained for $\alpha_1 = 0.6$, $\beta_1 = 0.3$, $\alpha_2 = 0.5$ and $\beta_2 = 0.45$. The right graph is obtained for $\alpha_1 = 0.3$, $\beta_1 = 0.2$, $\alpha_2 = 0.7$ and $\beta_2 = -0.3$. Both graphs use $v = 100$.

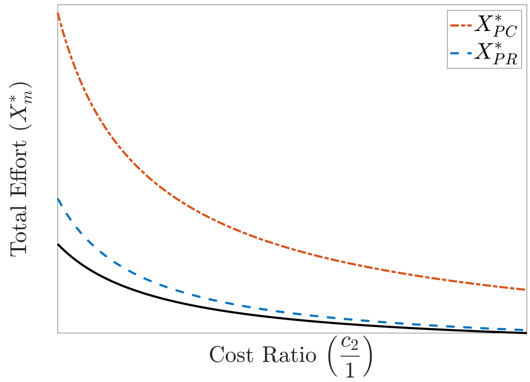
AC seems to predict considerably smaller ranges of equilibrium effort than Model AR. When players are inequality averse, both continua of equilibria contain the selfish Nash equilibrium. When one player is inequality prone[19] Model AR predicts higher total efforts in all equilibria as compared to Model AC. Like in the unique equilibrium cases, the incorporation of costs seems to reduce the amount of total efforts. Note though, that in the latter case both models predict overbidding relative to the selfish Nash prediction.

Given our analysis of the ex-post models in the previous sections, we have to conduct a comparison using numerical methods. Figure 3.3.14 shows how total effort evolves as the cost of player 2 increases. For all trials we ran, the qualitative implications are exactly as in this figure. If social comparison is based on ex-post comparison, the incorporation of costs increases total effort, as compared to the case where revenues are compared. This is, at least for all different cases we computed, irrespective of whether players are inequality prone, inequality averse or selfish. Also, the difference between the selfish total effort and the social preference models reduces more quickly, the less (more) guilt (spite) is in the model

[19] This is only possible to a certain degree when one wants to generate cases in which a continuum occurs in both models.



(a) Inequality Averse Players



(b) One Inequality Prone Player

Figure 3.3.14 : Total Effort in Models PR & PC for Increasing Cost Asymmetry

Note: Again, the black solid line represents the Nash equilibrium in case of two selfish players. The graphs are obtained for the same parameters as in the respective panels of Figure 3.3.13 . Qualitatively, there is no difference to allowing two inequality prone players.

(i.e., the less $\beta_1 + \beta_2$). The higher costs for player 2 have two effects on equilibrium play. On the one hand if player 1 is spiteful, she can rely on the high costs of her opponent to induce the AI that she enjoys and can focus on her own payoff, which she still enjoys more. On the other hand, if player 2 is spiteful, the outcome where she can enjoy AI, i.e. when she wins, becomes less and less likely. Again, reducing her effort becomes more attractive as a means to reduce costs, which are particularly high for her.

The standard result that total effort decreases with asymmetry thus still holds true irrespective of the model.

3.4 Conclusion

We have seen four different specifications of social preferences in probabilistic or share contests. While in probabilistic contests, the ex-post models seem to be more plausible, the ex-ante models have some convenient properties and seem to be applicable in share contests and probabilistic contests with strong cost asymmetries. As pointed out in the literature, the ex-ante specification is closer to norms of procedural fairness, while ex-post models rather model consequentialist fairness norms. It is not clear to us though, how this implies the incorporation or exclusion of costs. Although the inclusion of costs might be more plausible in some if not most contexts, we show that, concerning the qualitative implications, models with revenue comparison are equivalent and tend to be more tractable than their net-payoff counterparts. The ex-post models on the one hand invoke the more conventional use of expected utility and also allow (at least partially) to estimate social preference parameters from experimental data with standard techniques. Throughout the models cost asymmetry reduces total effort, but the strong player might not exert the highest effort when cost asymmetry is low under social preferences. This has implications for contest design when the high ability player is desired to exert high efforts.

A couple of further steps seem immediate. Hoffmann and Kolmar (2017) analyse a sequential contest game and consider the N-player game. Incorporating costs might introduce a further (dis-)incentive for the Stackelberg leader to stop the follower from exerting significant effort, depending on her social preferences. This might have implications for the order in which applicants are invited to job interviews when information about performance in these interviews is (at least partially) known to the other applicants.

We briefly discussed an existing empirical technique to estimate social preference parameters from experimental data and why it is problematic. From our own experiment, we have datasets on contest games in the lab, for symmetric cases. We also obtained secondary data from Fonseca (2009) and Kimbrough et al. (2014) that invoke different costs/impacts in the CSF. Putting the models to the test to see which performs better might be the practical equivalent of the exercise done here. In the absence of an empirical strategy, this might proof complicated though. This of course leads immediately to conducting a new experiment in case the data does not lend itself to test the above hypothesis appropriately.

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3.A Proofs

Proof of Lemma 3.2. By inspection of the utility function we see that the α part can only be relevant when $\Delta w_i < 0$, while the β part can only be relevant when $\Delta w_i > 0$. The respective parts of the best-response function are obtained by maximising the respective type's utility. The resulting best response function of the α type is

$$\arg\max_{x_i} U_i^{AC}(x_i|x_j \wedge \Delta w_i < 0) = \sqrt{A_i \frac{x_j}{c_i} v} - x_j$$

while for the β type we have

$$\arg\max_{x_i} U_i^{AC}(x_i|x_j \wedge \Delta w_i > 0) = \sqrt{B_i \frac{x_j}{c_i} v} - x_j,$$

Before we can use a diagrammatic approach to prove the shape of the best replies, we need the following lemma. Denote $\Delta_0(c_2)$ the function resulting from solving $\Delta w_1 = 0$ for x_1. This function depends on c_2.

Lemma 3.6.
For any $c_2'' > c_2'$, the we have $\Delta_0(c_2'') < \Delta_0(c_2')$.

Proof.

I proceed in two steps. First, I show that at $(0, 0)$, the slope of $\Delta_0(c_2'')$ is lower than $\Delta(c_2')$. Second, I verify that for any $(x_1, x_2) >> (0, 0)$, the above claim in the lemma holds.
At $(0, 0)$, both the prize is split according to the tie rule and each player's payoff is $\frac{1}{2}v$. Suppose player 2 has costs c_2'. If player 2 now increases her effort by some arbitrarily small $\epsilon > 0$, her payoff becomes $v - c_2'\epsilon$. Along Δ_0', Player 1 cannot increase her effort to ϵ as well as in that case $w_1 = \frac{1}{2}v - \epsilon > \frac{1}{2}v - c_2'\epsilon = w_2$. Thus, she increases it to some level $\epsilon' < \epsilon$, where player 2's higher share of the prize, but also higher costs, balance, such that $w_1 = w_2$. The continuity of all involved functions guarantees that such an ϵ' exists. Repeating this for c_2'' we see that this increase must be even lower than ϵ'. Thus, the slope of Δ_0 is positive, lower than 1 and decreasing in c_2.
The implicit derivative of Δ_0 with respect to x_2 is given by

$$\frac{\partial \Delta_0}{\partial x_2}(c_2) = \frac{2x_2 v - c_2(x_1 + x_2)^2}{2x_2 v - (x_1 + x_2)^2}$$

<u>Case 1: $2x_2 v - (x_1 + x_2)^2 > 0$</u>
Since the denominator is independent of c_2, we have $\frac{\partial \Delta_0}{\partial x_2}(c_2') > \frac{\partial \Delta_0}{\partial x_2}(c_2'')$ whenever

$$2x_2 v - c_2'(x_1 + x_2)^2 > 2x_2 v - c_2''(x_1 + x_2)^2$$

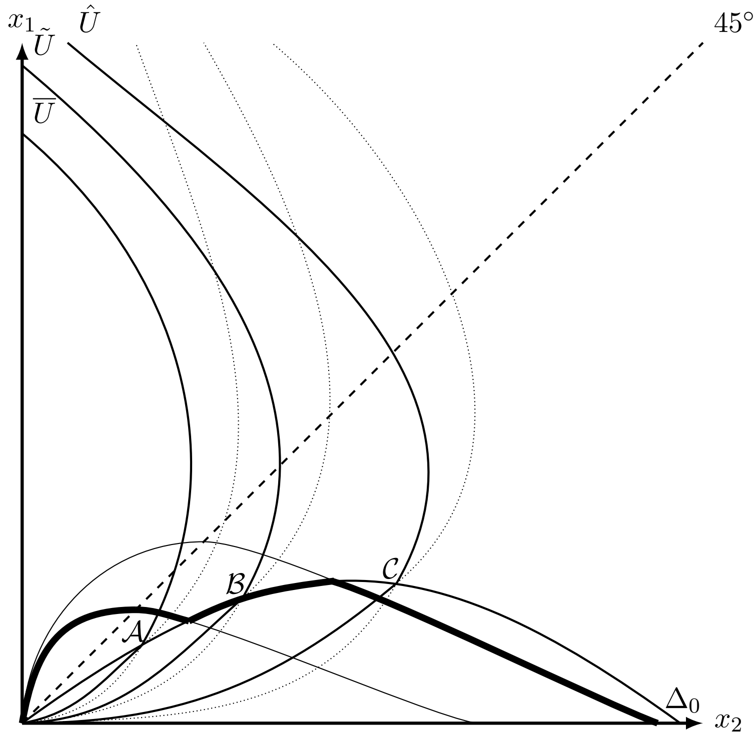

Figure 3.A.15 : No Profitable Deviations in the Best Response Function

Note: The thick solid line is player 1's best response function. The thin solid lines are player 1's indifference curves for the α-type and the β-type respectively, in the regions where they are relevant. The closer they are to the origin, the higher the level of utility. The grey shaded lines are their continuation in the region where the type is not active.

which is true whenever $c_2'' > c_2'$.

Case 2: $2x_2v - (x_1 + x_2)^2 < 0$

We have $\frac{\partial \Delta_0}{\partial x_2}(c_2') < \frac{\partial \Delta_0}{\partial x_2}(c_2'')$ whenever

$$2x_2v - c_2'(x_1 + x_2)^2 > 2x_2v - c_2''(x_1 + x_2)^2$$

which is true whenever $c_2'' > c_2'$. □

Consider any of the Figures 3.2.3 to 3.2.6 . Whenever each type's best response is in the respective region, this part describes the best response. Whenever the best response of the α type is in the β region and vice versa, one can show that choosing efforts on Δ_0 has no profitable deviation. Consider Figure 3.A.15 .

Each of the three dashed curves labelled $\overline{U}, \tilde{U}$ and $\hat{U}$ represents an indifference curve at a given utility level. The closer the curve to the left, the higher the utility level.[20] In the region above Δ_0, the indifference curve corresponds to the β-type's utility function, below it belongs to the α-type. The corresponding parts in each irrelevant region are shown in dotted lines.

At point $\mathcal{A}$, both types' best response functions are above $\mathcal{A}$. Thus, there exists a profitable

[20]Fixing the level of own effort, moving to the left just decreases the opponents effort and thus only increases chances of winning while keeping costs constant.

deviation by moving upwards. Since that leaves us in the region where player 1 receives the β-type's utility, this type's best response function is the optimal effort choice at that level of x_2. The same argument holds for a downward movement at point $\mathcal{C}$, leaving us in the region where player 1 receives the α-type's utility, making that type's best response the optimal choice at this level of x_2. For point $\mathcal{B}$, note that there is one type's best response above and below that point, respectively. An upward movement leaves us in the β region, while β's best response lies below $\mathcal{B}$. A downward movement leaves us in the α region, while α's best response lies above $\mathcal{B}$. Playing Δ_0 is thus a best response, since there is no profitable deviation. This exhausts all possible points on the best response functions for all different parameters.

Plugging each type's best response into $\Delta w_i = 0$ we get

$$x_j^\alpha = \frac{\left((A_i+2) - \sqrt{(A_i^2+4) - 4\frac{A_i}{c_i}}\right)^2}{4A_i(c_i+c_j)^2} c_i v \quad \hat{x}_j^\alpha = \frac{\left((A_i+2) + \sqrt{(A_i^2+4) - 4\frac{A_i}{c_i}}\right)^2}{4A_i(c_i+c_j)^2} c_i v$$

for the α type and

$$x_j^\beta = \frac{\left((B_i+2) - \sqrt{(B_i^2+4) - 4\frac{B_i}{c_i}}\right)^2}{4B_i(c_i+c_j)^2} c_i v \quad \hat{x}_j^\beta = \frac{\left((B_i+2) + \sqrt{(B_i^2+4) - 4\frac{B_i}{c_i}}\right)^2}{4B_i(c_i+c_j)^2} c_i v$$

for the β type. Note that $\hat{x}_j^\alpha > x_j^\alpha$ and $\hat{x}_j^\beta > x_j^\beta$. Also, these numbers might not be real in which case there is no intersection in the (x_1, x_2) plane.

The 'equal payoff' section of the best responses obtains from solving $\Delta w_1 = 0$ for x_1. Since this solution involves a quadratic it provides two equality curves. Fix some $x_2 = q$ that lies the range where player i plays the equal payoff effort. Moving vertically up to the upper equality curve, player 2's payoff decreases. Once we reach the upper equal payoff curve, there is still no deviation incentive regarding the two best response curves, but since $w_1 = w_2$ and we just argued that w_2 is lower at this point than on the lower equal payoff curve, this means that w_1 is lower there as well. This is not a deviation incentive and player 1 always chooses the minimum between the two curves. $\square$

Proof of Proposition 3.2. Consider the second unique equilibrium case first ($c_2 > \bar{c} > 1$). The strategies are obtained by solving the system of equations given by player 1's α-part of her best response functions and player 2's β-part. The important part is to verify that these two parts are actually relevant in equilibrium. Thus we need to investigate

$$\frac{x_1^*}{x_1^* + x_2^*} v - x_1^* - \frac{x_2^*}{x_1^* + x_2^*} v + c_2 x_2^* < 0$$

This reduces to

$$(B_2 - A_1)(A_1c_2 + B_2)^2 > A_1B_2(A_1 + B_2)(B_2c_2 - A_1)$$

This equilibrium lies under the 45°-line, thus $x_2^* > x_1^*$ which implies $B_2 > A_1$and thus $-\beta_2 > \alpha_1$.

The proof for the low asymmetry case ($c_2 > \underline{c} > 1$) is analogous.

If both intersections lie in the regions where the respective parts of the best response functions are irrelevant, they must have overlapping sections on the equality curve. The segment is between wherever the second β-curve intersects with, and the first intersection of one of the α-curves with $\Delta w_i = 0$. Formally, from perspective of player 1, this is

$$\left[\max(x_2, BR_2^{AC}(x_1^\beta)), \min(x_2^\alpha, BR_2^{AC}(x_1^\alpha))\right]$$

Player 1's strategies are the corresponding best responses, which for these values are guaranteed to lie on the $\Delta w_i = 0$-curve. □

Proof of Lemma 3.3. The interval $(\underline{c}, \overline{c})$ is nonempty, if and only if

$$\overline{c} = \frac{A_2}{B_1}\left(\sqrt{\frac{1}{4}(A_2 - B_1)^2 + 1} - \frac{1}{2}(A_2 - B_1)\right) \geq \frac{B_2}{A_1}\left(\frac{1}{2}(B_2 - A_1) + \sqrt{\frac{1}{4}(B_2 - A_1)^2 + 1}\right) = \underline{c}.$$

This inequality can be rearranged as

$$\left(1 - \frac{A_2}{B_1}\right)A_2 + \left(1 - \frac{B_2}{A_1}\right)B_2 + \frac{A_2}{B_1}\sqrt{\frac{1}{4}(A_2 - B_1)^2 + 1} - \frac{B_2}{A_1}\sqrt{\frac{1}{4}(B_2 - A_1)^2 + 1} > 0.$$

The difference between the two square root terms is lowest whenever the positive term attains a minimum and the term after the negative sign attains a maximum. Optimising both terms with respect to the two expressions A_1 and B_2 (A_2 and B_1 respectively) shows that the first order conditions are linearly dependent. They are given by the one equation

$$(A_2 - B_1)^2 + (A_2 - B_1)A_2 = -4$$

The second term on the left hand side attains a minimum at -4 for which the first term is strictly positive. Thus, there is no inner solution.

We need to investigate the corner solutions. For term one $\left(\frac{A_2}{B_1}\sqrt{\frac{1}{4}(A_2 - B_1)^2 + 1}\right)$ we have

	$A_2 = 1$	$A_2 = 2$
$B_1 = 0$	∞	∞
$B_1 = 2$	$\frac{1+\sqrt{5}}{2}$	1

while for the second term $\left(\frac{B_2}{A_1}\sqrt{\frac{1}{4}(B_2-A_1)^2+1}\right)$ we have

	$A_1 = 1$	$A_1 = 2$
$B_2 = 0$	0	0
$B_2 = 2$	$\frac{1+\sqrt{5}}{2}$	1

The minimum between those two terms thus attains for $A_1 = 1 = A_2 = 1$ and $B_1 = B_2 = 2$ or $A_1 = A_2 = 2$ and $B_1 = B_2 = 2$. In both cases, the difference becomes zero. Thus

$$\begin{aligned}&\left(1-\frac{A_2}{B_1}\right)A_2+\left(1-\frac{B_2}{A_1}\right)B_2+\frac{A_2}{B_1}\sqrt{\frac{1}{4}(A_2-B_1)^2+1}-\frac{B_2}{A_1}\sqrt{\frac{1}{4}(B_2-A_1)^2+1}\\ &\geq\left(1-\frac{A_2}{B_1}\right)A_2+\left(1-\frac{B_2}{A_1}\right)B_2\end{aligned}$$

If the right hand side is non-negative, this is a sufficient condition for $\bar{c} \geq \underline{c}$ to hold. That is the condition stated in the lemma. □

Proof of Corollary 3.2. For any positive constants (k, q), the total equilibrium efforts in the case of a unique equilibrium are given by

$$X_m^* = \frac{kq}{k+qc_2}v.$$

The derivative of that expression w.r.t. c_2 is given by

$$\frac{\partial X_m^*}{\partial c_2} = -\frac{kq}{(k+qc_2)^2}qv < 0$$

for any $(k,q) \in \{(a_i, b_j), (A_i, B_j), (d_i, d_j)\}$ for $i, j \in \{1, 2\}$ and $i \neq j$.
For player two, the general structure of her equilibrium effort is

$$x_{2,m}^* = \frac{k^2 q}{(k+qc_2)^2}v.$$

The derivative w.r.t. c_2 is given by

$$\frac{\partial x_{2,m}^*}{\partial c_2} = -2\frac{k^2 q}{(k+qc_2)^3}vq < 0$$

for any $(k,q) \in \{(a_i, b_j), (A_i, B_j), (d_i, d_j)\}$ for $i, j \in \{1, 2\}$ and $i \neq j$.
Player 1's equilibrium effort in these equilibria is given by

$$x_{1,m}^* = \frac{kq^2 c_2}{(k+qc_2)^2}v.$$

Taking the derivative, we get

$$\frac{\partial x^*_{1,m}}{\partial c_2} = \frac{k^2q^2 - kq^3c_2}{(k+qc_2)^3}v,$$

which is positive if $\frac{k}{q} > c_2$ and negative if $\frac{k}{q} < c_2$ for any $(k,q) \in \{(a_i, b_j), (A_i, B_j), (d_i, d_j)\}$ for $i, j \in \{1, 2\}$ and $i \neq j$.

Together with the conditions for uniqueness of equilibria in the ex-ante models, this concludes the proof. $\square$

Chapter 4

Multidimensional Identities in Conflict

Subhasish M. Chowdhury[‡] and Paul M. Gorny[§]

Abstract

Social Identity has been shown to induce strong effects on anti- or pro-social behaviour. This is particularly true when it comes to (violent) conflict. The amount of possible identities is too large to be tested individually in the lab. We propose a classification of identities into minimal (induced by the experimenter), horizontal (having no objective hierarchy) and vertical identities (having an objective hierarchy). In a lab experiment, we present information about the opponent's identity on either none, one or two of those identity dimensions. For the vertical identity, competition is more pronounced among individuals of the same stratum than among those of different strata. Other comparisons lack statistical significance but we list a series of qualitative findings. We document session effects that attenuate our results. A model of social distance structures most of the qualitative findings and points to design aspects for follow-up sessions.

Keywords: Conflict; Identity; Social Distance; Contest

JEL: C72; C91; D74

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4.1 Introduction

When we loosely speak about *social identity*, we typically mean a collection of personality traits, beliefs, visual characteristics and potentially labels that define who we are as individuals or groups and how we compare to other individuals or groups. The degree of (dis)similarity in those traits that are salient in a given situation, provides us with (sometimes flawed) information on how to interact with another person or group. This can be due to individual experiences attributed to that trait, stereotypes existing in a social group or society as a whole and social norms on how to treat individuals from within or outside one's own group.

Thinking about violent conflict of any scale, the conflicting parties must assign some identity to themselves that is different from the identity of their 'enemies' in terms of traits that are relevant (salient) to the context of the conflict. If fans of rival football teams clash in the streets, this might be club membership. In everyday conflicts between individuals in Jerusalem, it is rather religion, ethnicity, language or all of them in at once. When individual combatants meet in interstate war, it is their nationality. The individuals' social identity allows for a categorisation into in-group and out-group by aggregating all the salient characteristics. A particular aspect of conflict is, that such a distinction is typically sharp and dichotomous, as others are divided into 'friend' or 'foe'. These categories are also persistent, as they can still be relevant to individuals, even in the absence of their own group. It is important to understand how these categorisations occur. They can have an impact on people's social and economic welfare, for example, when in-group favouritism and out-group hatred, extend to large-scale conflict like civil war and ethnic cleansing.

We want to investigate, whether a typology of these identities exists, which of them are considered salient and how they affect behaviour in an experimental conflict game. We present information on different amounts and types of identities to the contestants across different treatment groups and compare the amount of effort they exert against each other. We then structure our findings theoretically, by incorporating social distance and status concerns in a model of social identity that is then applied to contest. The main questions we want to pursue here are twofold. First, we want to see, whether there is a way to classify certain types of identities by their (common) effects on conflict behaviour. Second, we investigate, whether making multiple identities salient can change conflict intensity in a systematic way.

One part of the classification we attempt is based on Sen (2007) who points out that there are different strengths of identity depending on how much they are based on experience and factors that the individual perceives as 'real'. He thus distinguishes three types of identities. *Classifications* simply assign individuals arbitrarily to groups. *Minimal identities* are assigned according the minimal group paradigm (Tajfel, 1970) by, for example, grouping them according to the colour of their clothes or their preference for abstract paintings. *Real identities* are traits and attributes that individuals are aware of and carry with them in all-day life. While classifications have shown to not have any effect in this type of experiments (Chowdhury et al., 2016), we suggest yet another distinction within the real identities. Ethnicity, language and religion are examples of what we coin *horizontal identities*. They do not have an obvious ranking for an impartial third-person observer. A member of a certain group, based on these identities might feel superior to out-groups. But it is possible, and often likely, that this is a mutual view and members of the other groups feel superior too. Social strata, casts and job seniority are examples of *vertical identities*. They can clearly be ranked by an impartial third-person observer and the members of the different groups likely agree, at least to a certain extent, to that ranking. Even if they do not, they are at least aware of this perception in society. We want to know, whether this difference in perception of the other group triggers systematically different behaviour in conflict.

Concerning our investigation of multidimensional identity, the Bengal Language Movement in the early 1950s is a prime example of how one identity dimension can counteract the effects of another. After India was partitioned into the dominion of Pakistan and the Union of India, East Pakistan belonged to the former. The population was roughly composed of two thirds of Muslims and one third of Hindus and the relation between these groups was notoriously tension-filled. At the same time the vast majority of the population belonged to the ethnic group of the Bengalis which was most strongly pronounced through their common language Bengali. In 1947, the Dominion of Pakistan decided on a resolution to make Urdu the official language in both Western and Eastern Pakistan. This led to demonstrations and riots against this bill in which all religious groups where involved (Rahman, 2002).

This is most striking, as the above-mentioned tensions between Hindus and Muslims triggered extreme acts of violence before. It seems, as if the commonality in ethnicity, and especially in language, outweighed the disparity in terms of religion. All of this happened because of an attentional shift induced by the government in Karachi. With their bill they made language a salient feature of the conflict and put religious aspects in the rear.[1] We investigate, whether a similar combination of identity dimensions can reproduce the core of this phenomenon in the lab. This is particularly important if we view social identity (at least partially) as a social construct that can be shaped by individual choices and corporate, societal or political values. We want to investigate, which type of dimensions should be made

[1] Many historians see this chain of events as the foundation for the liberation of Bangladesh in 1971.

more salient to reduce conflict on one or more of these levels.

The terms 'in-' and 'out-'group immediately convey an image of proximity/distance to the members of either group. We typically care more for people that are 'close' to us, while we are indifferent towards people we feel 'detached' from. We even 'distance ourselves' from those that hold values or have characteristics we oppose. The concept of *social distance* is the most natural modelling tool to capture this. We use it, to formulate a model that bases social preferences on social distance. Each individual's social 'location' is determined by her traits. Since social identity needs to be sufficiently strong to induce an in-group with associated out-group, this concept works, both, when such a categorisation is present, and when proximity between individuals does not lead to groups, based on social identity.

We find that making the vertical identity salient alone, increases effort relative to the other treatments and control. Being in the same level of the vertical identity seems to increase efforts more strongly than being in different strata. While this is due to high and low-level subjects, the high level subjects seem to have higher efforts on average across treatments. French students and females also have significantly higher efforts in this experiment. Although the design allowed us to run many interesting constellations of identities, the other treatments, particularly the ones with multiple identities, remain insignificant due to a low number of independent observations, relative to the high number of treatments.

Our model introduces social distance into the standard two-player Tullock (1980) contest. Low social distance below a threshold triggers altruistic behaviour, while distance beyond that threshold triggers spite. The vertical identity triggers a loss function that relates to social status. It predicts some of the above findings and most of the qualitative implications of the experiment and points at further questions to investigate.

Related Literature: The term social distance has been coined early in sociology (see e.g. Park (1924), Bogardus (1928) and Bogardus (1933)), in social psychology (e.g. Trope et al. (2007) and Brewer (1968)) and has been used since across the social sciences and in economics in specific. Hoffman et al. (1996) state that different perceptions of social distance to the experimenter changes the behaviour of subjects in the lab. They find that reduced social distance increases the offers in a dictator game significantly and they explain this by subjects conforming to norms they learned in their social environment outside the lab. A famous formalisation can be found in Akerlof (1997). In his model, the position of players on a line can be altered by the actions they choose. These actions directly affect utility but there are also positive externalities from being close to others. The model accounts for status seeking, i.e., going for the optimum behaviour in terms of the selfish utility. It also accounts for conformist behaviour, i.e., behaviour where all players choose the same position. It is possible to have both in equilibrium. While this model allows for mutually beneficial and selfish behaviour that does not explicitly aim at harming others, our model allows for spite. Such agents enjoy hurting their opponent because of the great social distance they have. Further experimental investigations in social distance in economics are due to Dufwenberg and Muren (2006), Buchan et al. (2006) and Ahmed (2007). The latter two find more pro social behaviour when social distance is low. Dufwenberg and Muren (2006) in a giving game find the opposite effect if social distance is equated with 'anonymity' (here: whether the recipients received their money in private or not), which the authors strongly warn against.

We relate social distance to social identity, which was first researched in social psychology. Muzafer Sherif is often referred to as the father of this discipline. In Sherif et al. (1961), the authors report on a series of famous experiments conducted in 1949, 1953 and 1954, commonly known as the Robbers Cave Experiments. In these experiments young boys were randomly divided into two groups (camps) that were physically isolated from each other and competed in a series of contentions. After a short time the boys in each group started to exhibit strong hostilities. These could, at least partially, be overcome by setting the groups a problem that they are unable to solve without the other group.
These results led Tajfel and Turner (1979) to formulate what is now known as social identity theory, defining three main mechanisms of identity driving individual behaviour. These are social identification, social categorisation and social comparison. In the context of social distance, these are components of the implicit measurement of social distance, which incorporates knowing your own location, the other individuals' location and how to form a measure of distance.
Studies on the relationship between social identity and social preferences in social psychology (e.g. Orbell et al. (1988) and Kinzler et al. (2009)) demonstrate that individuals behave strongly in favour of their own group, a behaviour known as in-group favouritism.

In the recent years, economics has been influenced by the above and more recent findings on the effects of social identity on behaviour. Akerlof and Kranton (2000) and Akerlof and Kranton (2010) plead for the advent of *Identity Economics* and list and model a series of behavioural anomalies and real world phenomena that cannot be explained without invoking identity in the analysis. In experimental economics, a recent seminal article by Chen and Li (2009) has documented some of these effects for strategic behaviour in a standard set of economic games (see Charness and Rabin (2000)) in the lab. This study documents significant effects of group membership on social behaviour. Still, the aspects of reductionism with respect to identity and its interplay with conflict have not yet been addressed in an experimental study. We intend to add to the literature by specifically considering three additional aspects in our project.

First, the set of sequential games used by Chen and Li (2009) misses an important aspect in situations of conflict and fierce competition. Efforts are exerted to increase the chance of winning, but since the outcome of the conflict is still not fully deterministic, contestants can end up incurring the costs for their effort, even when they lose. Therefore, we suggest to shed light on the behavioural effects of social identity in a contest game in the lab and in a stylised model. Second, as has been pointed out by Sen (2007), identities differ qualitatively in terms of their effect size on conflict. We suggest different types of real identity to structure these differences. Finally, we explore how adding multiple salient identity dimensions changes behaviour relative to only making a single identity dimension salient. The studies that are closest to ours in terms of our experimental design are Chowdhury et al. (2016) and Hong et al. (2016). While the former uses a contest game as well, its focus is on group contest and one-dimensional identity without our classification of identity types, the latter makes these distinctions and analyses multidimensional identity but on a different type of game.

The rest of the paper is organised as follows. Section 4.2 introduces the experimental design. The analysis of the experimental data can be found in section 4.3. Section 4.4 presents our model and discusses how it explains some of the qualitative findings of the experiment. Section 4.6 concludes.

4.2 Experimental Design

In our experiment, subjects receive a certain amount of information about their opponent's identity. This can either be no information, information on one identity or information on a combination of two identities. We use three different types of identity. The first two of these are real identities. A specific school degree that allows French students to enter university is the CPGE (*Classes Préparatoires aux Grandes écoles*) that also allows entry into the prestigious Grandes Écoles like the École Polytechnique or HEC Paris. It is a highly selective and expensive 2-3 year course. If students do not obtain this distinction, they are admitted with their regular, typically public high school degree, which can be denoted with AST (*Admission Sur Titre*). Burgundy School of Business (from hereon BSB) Dijon, the site of the lab, allows students with both degrees and the amount of students from both backgrounds is relatively balanced with a slight skew towards CPGE students (a summary can be found in Table 4.2). Students are extremely aware of this identity as it is relevant in a lot of everyday situations and a common characteristic to ask for when getting to know new peers.[2]

As a candidate for a horizontal identity we use the fact that BSB offers two language tracks

[2]We are aware that the CPGE students in BSB are the ones that did not get into a Grand École and that this might affect results. We are trying to mitigate the effect of negative feelings toward their own degree through rejection by eliciting an implicit measure for subjects' attitude towards the CPGE.

within each degree. Students in the *parcours francophone* are taught entirely in French, while those in the *parcours anglophone* are taught entirely in English. Students only visit classes within their language track and thus this identity is salient as well. Additionally, we display instructions to subjects in the language that corresponds to their language track. As opposed to the school degree, the language track is not a group that existed before uni and students in each track could just be friends rather than a group with a joint social identity. Still, the concept of social distance would apply in that case and we argue that it should be closer for individuals from within the track than for individuals from the other track.

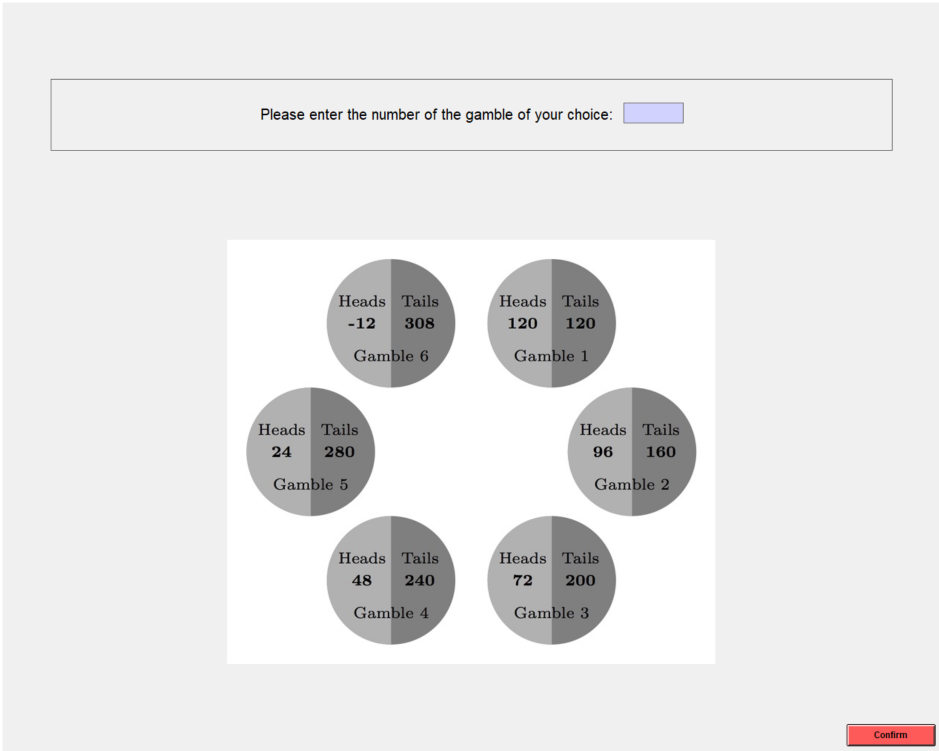


Figure 4.2.1 : Screenshot of Stage I

Finally, we create a minimal identity in the lab. To induce it, there are 5 rounds of pretreatment (stage I) in which subjects each have to decide which painting of either Paul Klee or Wassily Kandinsky they prefer.[3] Based on their decisions, subjects in each session are divided into a Klee and a Kandinsky group according to the count of Klee/Kandinsky paintings relative to the median value of choices. The painting identity is borrowed from Chen and Li (2009) and has, to our knowledge, first been used in Billig and Tajfel (1973) and several other studies since then.[4]

In stage II we elicit a risk measure according to Eckel and Grossman (2008). Subjects are given 6 gambles, which are clearly ranked in terms of their risk-neutral expected payoffs. Subjects can choose a gamble and the corresponding payoff is added at the end of the session

[3]These are: A Gebirgsbildung, 1924, by Klee, IB Subdued Glow, 1928, by Kandinsky, 2A Dreamy Improvisation, 1913, by Kandinsky, 2B Warning of the Ships, 1917, by Klee, 3A Dry-Cool Garden, 1921, by Klee, 3B Landscape with Red Splashes I, 1913, by Kandinsky, 4 A Gentle Ascent, 1934, by Kandinsky, 4B A Hoffmannesque Tale, 1921, by Klee, 5A Development in Brown, 1933, by Kandinsky, 5B The Vase, 1938, by Klee.

[4]See for example Gaertner and Insko (2000), Platow et al. (1997) and Petersen and Blank (2003).

Identity Dimension	None	Language		School	
		Same	**Different**	**Same**	**Different**
None	Control	SL	DL	SS	DS
Same Painting	SP	–	SPDL	–	SPDS
Different Painting	DP	DPSL	–	DPSS	–

Table 4.1: A 3x5-4 Experimental Design

by using a random draw made by the computer. While it is technically possible to end up with a negative payment due to the riskiest gamble, no subject actually ended up in this situation.[5]

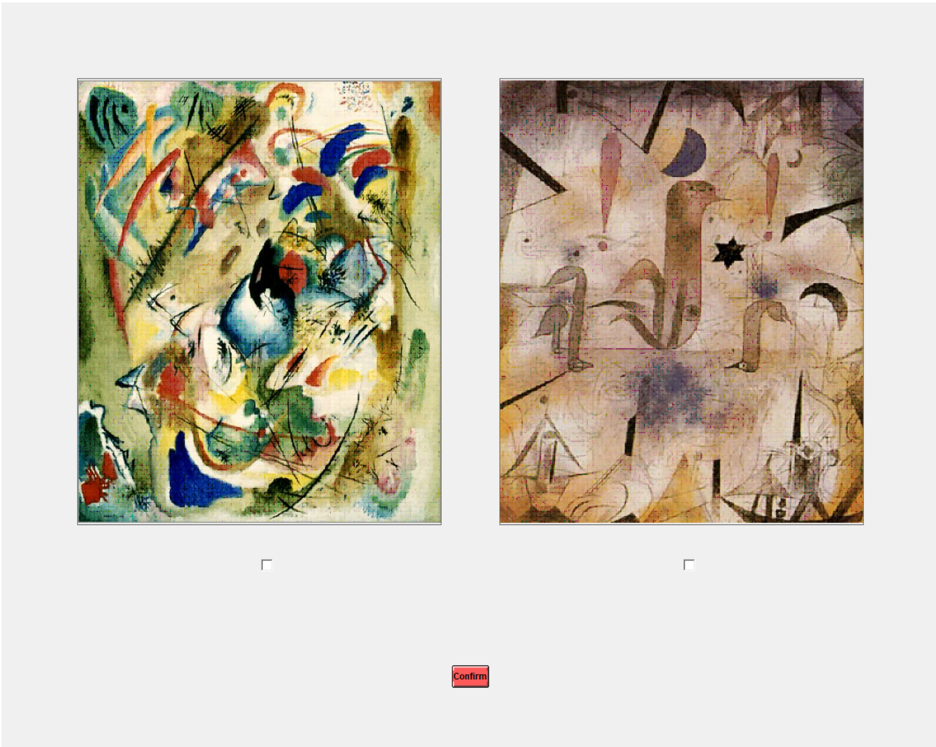


Figure 4.2.2 : Screenshot of Stage II

We have a total of 15 sessions. Except for two sessions, the session size was between 12 and 20 subjects.[6] We use a partner matching protocol. Subjects are matched randomly at the beginning of each session into pairs that are stable for all periods. In stage III, subjects play 25 rounds of individual contest, 5 of which are eventually payoff relevant for all subjects in the session and these are chosen at random. A contest is a game in which costly efforts are exerted to increase the chance of receiving a prize of fixed value. Subjects have an endowment of 160 Experimental Currency Units (ECUs) each round to exert this effort and cannot carry over ECUs to the next round. The probability of winning is given by

$$\frac{\text{Own Bid}}{\text{Own Bid} + \text{Opponent's Bid}}.$$

[5]The extreme values of final payments are 5.13€ and 16.13€.
[6]Session 2 has 36 subjects while session 12 has only 4 subjects.

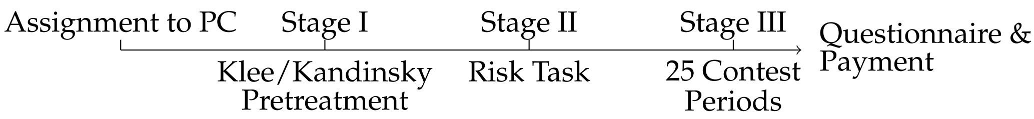


Figure 4.2.3 : Timing of Experiment

In case of a tie a fair coin flip decides. The winner is determined by the computer according to the above probability and is awarded a prize of another 160 ECUs. The full timing of the experiment can be seen in Figure 4.2.3 . The exchange rate is 100 ECU to 1€. The average final payoffs in the experiment are 10.46€.[7]

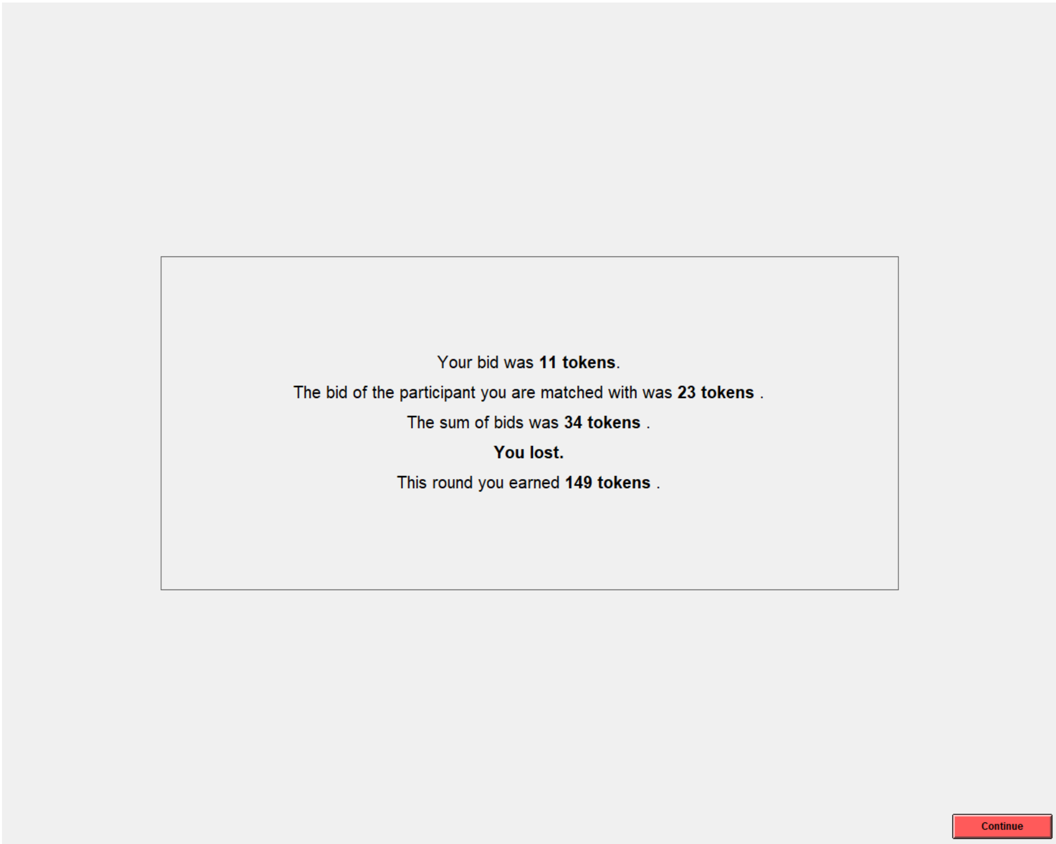
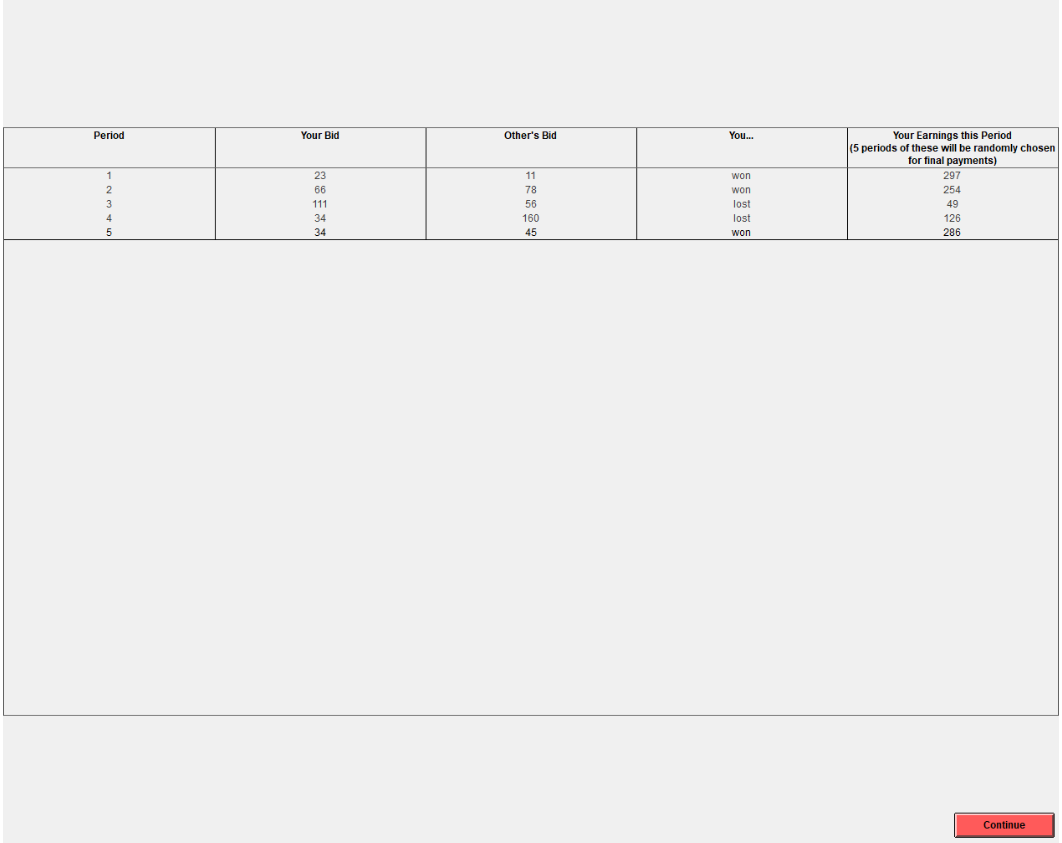


Figure 4.2.4 : Feedback on Previous Round

Subjects were given feedback about their own effort, their opponent's effort, the winner of the round and their own payoff in this round.

Since in real world situations it is unlikely that individuals in a conflict do not know their opponents' identity on the dimensions that are salient, we only consider matchings where information is symmetric. From hereon we refer to language as the horizontal identity and school as the vertical identity. The entries in each cell of Table 4.2 indicate the number of subjects for each identity. In total we invited 224 subjects. Summary statistics can be found in table 4.3. For each subject, we have their minimal, horizontal and vertical identity. The

[7]Assuming symmetry, risk neutrality and using the Tullock contest success function, the resulting expected payoff is $\pi\left(x_i, x_j\right) = v x_i/(x_i + x_j) - x_i + m$ where x_i is player i's effort, v is the value of the price and m is the individual's endowment. Thus, the expected payment for each subject in each session is 10€ plus the expected rational choice in the risk task (1.52€), totalling 11.52€.



Period	Your Bid	Other's Bid	You...	Your Earnings this Period (5 periods of these will be randomly chosen for final payments)
1	23	11	won	297
2	66	78	won	254
3	111	56	lost	49
4	34	160	lost	126
5	34	45	won	286

Figure 4.2.5 : Feedback on all Previous Rounds

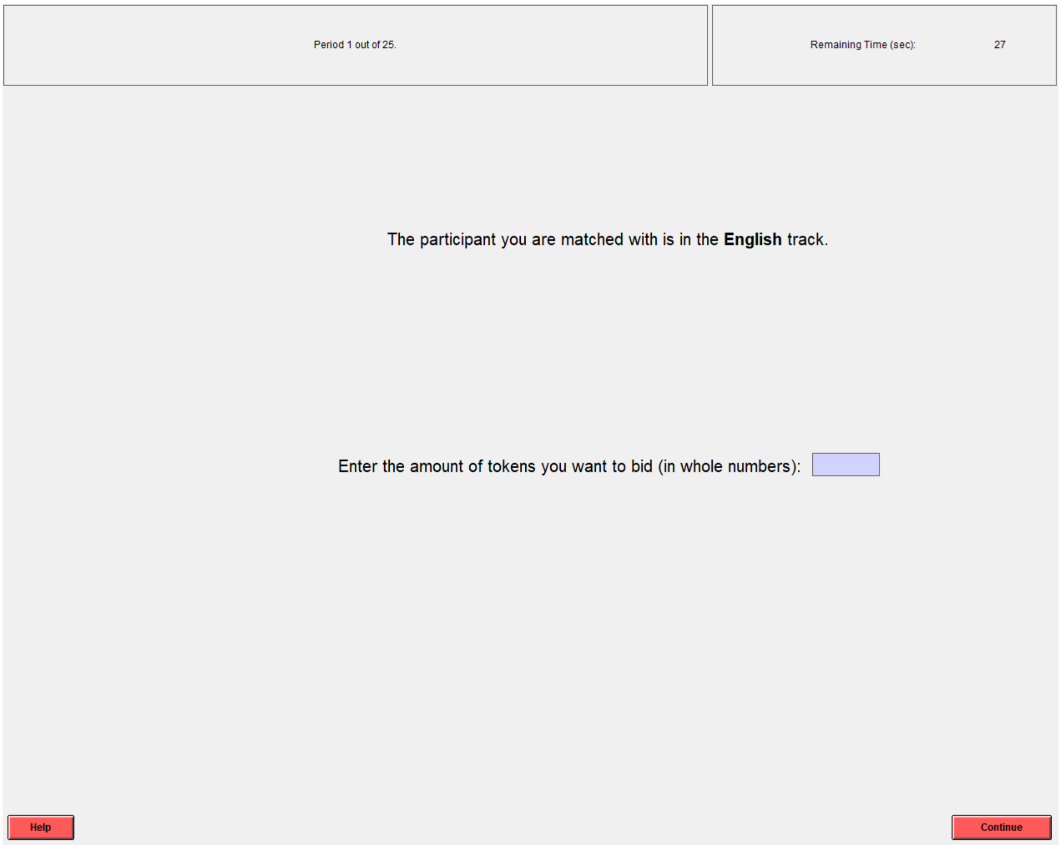


Figure 4.2.6 : Screenshot of Stage II (Horizontal Treatment)

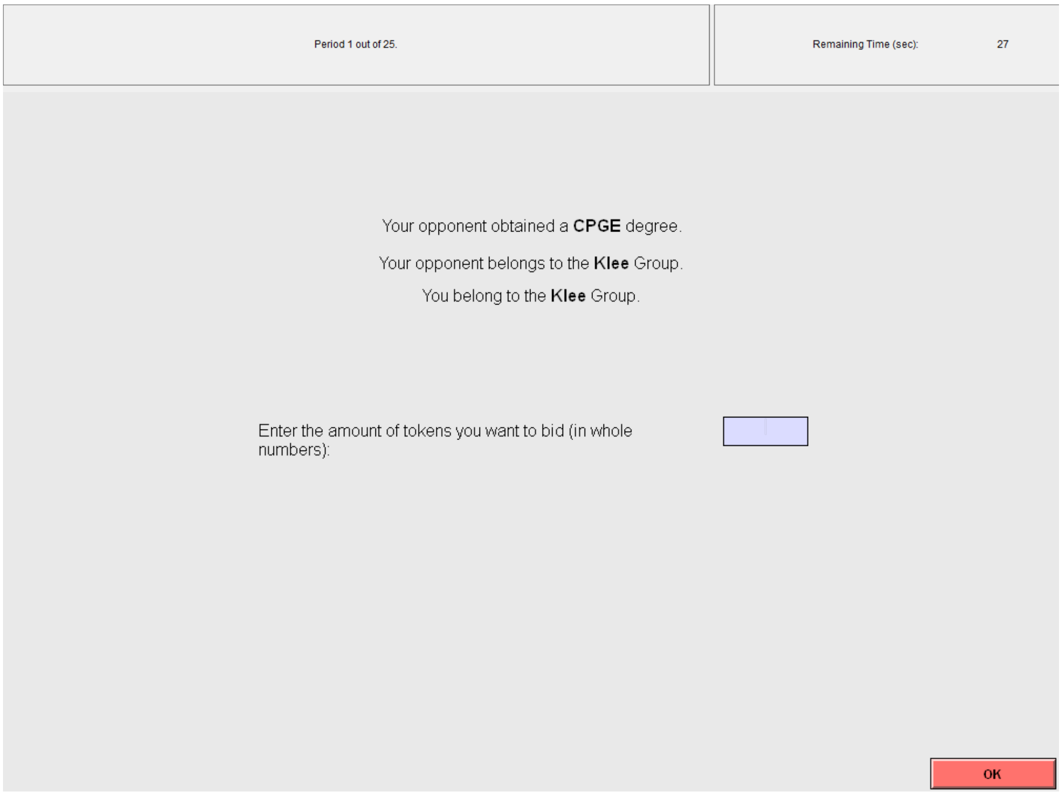


Figure 4.2.7 : Screenshot of Stage II (Vertical + Minimal Treatment)

Minimal		Horizontal		Vertical	
Kandinsky	**Klee**	**English**	**French**	**AST**	**CPGE**
112	112	103	121	101	123

Table 4.2: Number of Subjects for Each Identity

Variable	Mean	Std. Dev.	Min	Max
Female	0.502	0.500	0	1
Effort	64.750	40.171	0	160
Language	0.543	0.498	0	1
School	0.443	0.497	0	1
Age	20.393	1.090	18	25
Risk Aversion	3.589	1.898	1	6
Final Earnings	10.459	2.465	5.13	16.13
Observations		219		

Table 4.3: Summary Statistics

identity/identities that is revealed to their opponent is/are randomly chosen at the beginning of each session by the computer. Instructions are displayed according to the language they are taught in, irrespective of the revealed identity.
Previous results in Chen and Li (2009) and Chowdhury et al. (2016) have shown that differences in group identity increase effort/defection (decrease cooperation) relative to treatments with in-group members. From this we derive two hypotheses concerning the single identity treatments concerning the minimal and the horizontal identity.

Experimental Hypothesis 1.

- *Effort in DP > Effort in SP (Chen and Li, 2009)*
- *Effort in DL > Effort in SL (Chowdhury et al., 2016)*

Differences should spur effort contributions while commonalities should reduce them as subjects are expected to fight harder with opponents from their out-group as compared to their in-group.
For the remaining comparisons, our study remains explorative and the results are intended to inform the model that attempts to structure the findings.

4.3 Results

In table 4.4 we see means of efforts across the different treatments. It is visible that, given the high variances and low number of subjects in each treatment, many of the comparisons are not significant.[8] In general we can state though, that making (solely) the vertical identity salient increases effort levels relative to treatments in which only the horizontal identity is revealed. Similar findings can be made with respect to the comparison of the vertical and the minimal identity, which is also horizontal. Beside the DPSS treatment, revealing multiple

Control	**SL**	**DL**	**SS**	**DS**	**SP**	**DP**
65.52 (37.14)	59.93 (31.29)	57.06 (34.27)	82.13 (49.90)	68.80 (39.24)	52.12 (34.92)	60.39 (32.85)

SPDL	**SPDS**	**DPSL**	**DPSS**
67.78 (48.15)	68.85 (47.19)	67.54 (37.11)	57.16 (39.23)

Table 4.4: Means of Effort Across Treatment
Note: Standard Deviation in Parenthesis

identity dimensions to the participants seemed to slightly increase efforts relative to the

[8]As we are testing multiple comparisons, we should run a correction procedure like Bonferroni or Sidak (see Abdi (2007) for a summary and comparison). The naive tests we are running here should again be understood as an indication for future studies in terms of interesting hypothesis and number of subjects.

control. Relative to the level of efforts, the variance seems to be higher in the treatments SPDL, SPDS and DPSS. In general, the summary statistics already suggest, that inferences from the treatments where we induce multiple identity dimensions are hard to make.
We first present some non-parametric tests, then contrast them with regression results and finally investigate session effects that seem to be present in the data.

4.3.1 Minimal vs. Real and Horizontal vs. Vertical Identity

Conventionally, it is assumed that minimal identities induce smaller effects than real identities as they are newly created in the lab. Subjects thus do not feel this identity as strongly as the real identities that matter in every day life. This is in line with Chowdhury et al. (2016). In terms of horizontal and vertical identities, there is neither much previous evidence, nor an obvious conjecture of how they should compare.[9] Consider Figure 4.3.8 . The overall

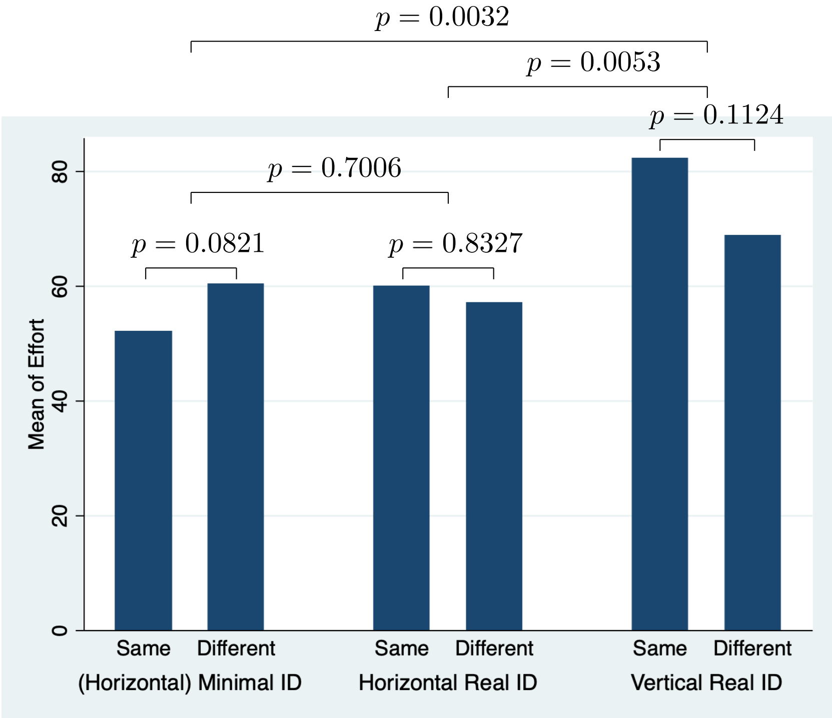


Figure 4.3.8 : Mean Efforts across Real and Minimal Identity Treatments
Note: p-values correspond to Mann-Whitney-U tests

difference between the minimal and horizontal identity is insignificant (p=0.7006).[10] The differences between minimal and vertical identity and horizontal and vertical identity respectively, are significant ($p = 0.0032$ and $p = 0.0053$ respectively) as is also visible from the graph. The comparison between the two treatments within each of the real identities is not significant and is displayed in the graph. If anything, the vertical identity can be suspected

[9] As the minimal identity in our experiment does not have an obvious ranking, it is also a horizontal identity in terms of the other classification.

[10] We are using sums of efforts across all periods, within each matching and for the painting and language treatments respectively for the test (i.e., SP+DP vs SL+DL). The p-value corresponds to a Mann-Whitney-U test.

to show a significant difference once more follow-up sessions are run.

The effort distributions for the minimal treatments differ significantly at the 10% level, while they do not for the real identity treatments, it seems that real identity has less effect on behaviour than minimal identity. In light of existing research this is unusual, particularly given the strong salience the language identity should have.
Pooling all observations for the horizontal and the minimal identity respectively shows no difference in efforts between them.

4.3.2 Multiple Identity Dimensions

We compare the single identity dimension treatment with those that display two dimensions to the subjects (one real and one minimal). We see that moving from SP to any treatment where we add a difference in a real identity, efforts increase independently of which identity dimension we choose.[11] These results lack statistical significance though. The case of adding

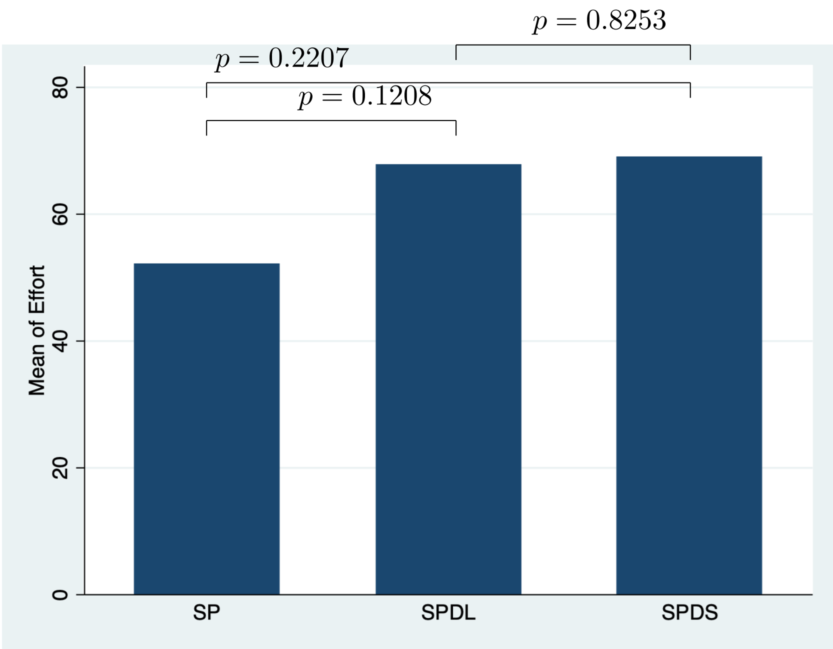


Figure 4.3.9 : Multiple Identity Dimensions relative to SP
Note: p-values correspond to Mann-Whitney-U tests

a similarity when facing a difference in the minimal identity is less clearly signed. While, in terms of efforts, we have $DP < DPSL$, we also observe $DP > DPSS$. Particularly given the earlier observation that efforts in the SS treatment are higher than in the other similarity treatments, this is puzzling.
The variance in the SPDL and SPDS treatments is higher than the one in DPSL and Control, while the means are fairly similar, but due to the small number of independent observations,

[11]We chose the comparison with SP and DP respectively as it allow for a more neat representation in two graphs.

these comparisons are insignificant, particularly since efforts are not normally distributed in most treatment groups. Still, an explanation for why this is the case could be the bigger

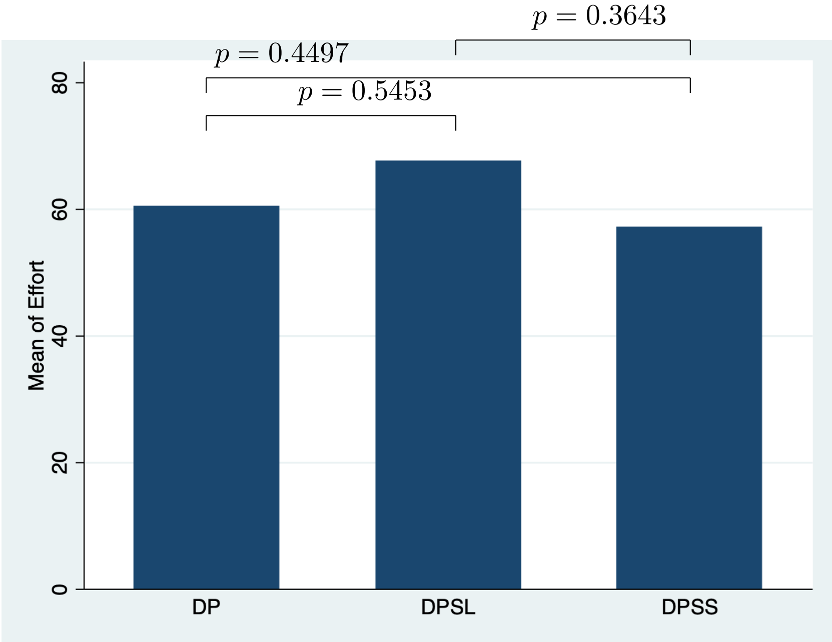


Figure 4.3.10 : Multiple Identity Dimensions relative to DP
Note: p-values correspond to Mann-Whitney-U tests

amount of information that is leading to confusion with the subjects. While an attentional shift from religion to ethnicity/language in the Bengal language movement clearly pointed towards one identity dimensions and drove out the other, subjects in this experiment might find it hard to decide which dimension to act upon. The fact that variances in the multidimensional treatments are slightly higher relative to their respective mean, could support this and even if statistical power is insufficient, the data at hand cannot reject this explanation.

A summary of all relevant treatment comparisons can be found in table 4.5.[12]

	SL	DL	SS	DS	SP	SPDL	SPDS	DP	DPSL
DL	0.8327								
SS	0.0290	–							
DS	–	0.0963	0.1124						
SP	0.1053	–	0.0082	–					
SPDL	–	0.3272	–	–	0.1208				
SPDS	–	–	–	0.5676	0.2207	0.8253			
DP	–	0.7055	–	0.2568	0.0821	–	–		
DPSL	0.4386	–	–	–	–	0.9349	–	0.5453	
DPSS	–	–	0.0413	–	–	–	0.3691	0.4497	0.3643

Table 4.5: P-Values of Mann-Whitney-U Tests

[12]See footnote 8 on multiple comparisons again.

4.3.3 Regression Analysis

Controlling for a number of demographics and answers to a questionnaire, the results do not change too much qualitatively.

Whether an identity is important to the individual or not should change the size of the effect that identity has on behaviour. Ideally, we would like to have subjects that strongly identify with the group sharing their respective identities. Since this might not be the case for all subjects and all identities we elicit and induce, we would like to control for the subjects' attitudes toward these identities. To get some proxies on these attitudes, we elicit answers to indirect questions in the post-experimental questionnaire. We ask for whether students think the CPGE is 'too expensive', 'correctly priced', 'too cheap' or whether they 'do not know'[13] with the variable WTPCPGE.[14] We also asked students for whether they are interested in arts (ARTS) and whether they watched TV predominantly in French (TVLang - French), English (statistical baseline) or do not have a preference (TVLang - Neither).

Consider Table 4.6. The relative ranking of the 'same' and 'difference' treatments for both real identities is unchanged for all specifications. In the vertical identity, both, similarities and differences increase efforts relative to all other treatments, but only SS is significantly different from the baseline. This vanishes as we control for WTPCPGE.[15] Within the minimal identities, both treatments have lower efforts than the baseline, but there is no statistically significant difference. Their relative ranking is as expected as, in terms of effort, DP>SP. The difference between SL and DL is statistically insignificant as well, and is also not large in magnitude.
It is interesting to note that CPGE students seem to be significantly more competitive than AST students throughout specifications.

Result 1.
The vertical identity induces higher effort contributions than the horizontal identity and the minimal identity.[16] *Students from the CPGE exert more effort than AST graduates.*

The most plausible explanation is one of selection in which more competitive students sort into the CPGE and thus bring this competitiveness into the lab. A competing hypothesis is that CPGE students might simply perform better than AST students. A better performance,

[13]The statistical baseline is 'No Response' as a technical issue in the first session with 10 subjects resulted in the loss of their responses.

[14]We tagged this variable WTP as in willingness to pay as it is easy to remember. This is due to the how the question was asked in the questionnaire. We are not assuming that it actually measures a willingness to pay or falls within that framework.

[15]We included those controls separately to make sure which control attenuates the effect of SS.

[16]The p-values of an F-test for the linear hypothesis that $\beta_{SS} + \beta_{DS} = \beta_{SL} + \beta_{DL}$ are 0.0034, 0.0039, 0.0059 and 0.068 for the four models respectively, and for $\beta_{SS} + \beta_{DS} = \beta_{SP} + \beta_{DP}$ they are 0.0013, 0.0011, 0.0014 and 0.0044.

under the assumption that subjects want to maximise their earnings, can be measured by their success to increase these. We should thus find higher earnings among the CPGE subjects than among the AST subjects. With the measures available, this cannot be supported. We run the same models with final earnings as a dependant variable and AST is insignificant (and positive, thus running counter to this explanation). This measure is not directly measuring the subjects' ability. It is only a hint that the selection story is more likely to explain the higher bids made by CPGE students. Future studies might want to control for a measure of cognitive ability like an IQ measure or the result of a raven's matrix test. Students in the French track exert significantly higher efforts than those in the English track. Controlling for the subjects' attitude towards language with TVLang slightly increases this effect.

Result 2.
French students exert higher efforts than English students.

Although we talk about language as a horizontal identity, we did not run the experiment in a language-free context. The site of the lab is located in France and this might have created some ranking that both English and French Track students implicitly agreed on. Additionally, students in the English track also had their instructions displayed in English and some of them struggled to understand certain words.[17] This lack of understanding could have led to a more cautious choice of efforts.

Partly consistent with Chowdhury et al. (2016), we find that females are bidding more aggressively than males in this contest experiment, holding treatment constant. From the descriptives on risk aversion we can already see, that females are more risk averse than males in this subject sample. Thus, the significance and magnitude of Female attenuates drastically if we exclude Risk Aversion.[18] Within one risk aversion level, females exert higher efforts than males.

4.3.4 Session Effects

With the large number of treatments and the relatively low number of subject pairs within each treatment, the experiment should be understood as an exploratory pilot that guides the design of future studies and gives more clear-cut predictions (either via the results found above or by the model that is generated from these first insights). The treatments are applied on the matching level and every treatment can be conducted in each of the sessions on one or more pairs of subjects. Since we wanted to obtain a balanced data set in terms of

[17] The vast majority of questions concerned the word 'gamble' in the risk task, but we cannot rule out that despite students are not asking questions in the contest rounds, they have trouble to understand aspects of it.

[18] If we exclude Risk Aversion from model (3), for example, the coefficient of Female changes to 1.868383 with a z-score of 1.57.

Dep.Var.: Effort	(1)	(2)	(3)	(4)
SL	-5.593	-6.229	-3.237	-6.287
	(-0.66)	(-0.75)	(-0.39)	(-0.76)
DL	-8.462	-7.794	-8.373	-6.820
	(-0.97)	(-0.92)	(-0.98)	(-0.81)
SS	17.57*	15.70*	15.96*	12.45
	(1.95)	(1.79)	(1.80)	(1.42)
DS	8.127	8.363	9.092	9.804
	(0.87)	(0.92)	(0.99)	(1.09)
SP	-13.40	-13.79	-12.49	-11.10
	(-1.53)	(-1.62)	(-1.46)	(-1.32)
SPDL	2.259	-0.378	1.674	2.877
	(0.25)	(-0.04)	(0.19)	(0.33)
SPDS	7.367	6.970	7.360	8.544
	(0.79)	(0.77)	(0.80)	(0.94)
DP	-5.132	-5.677	-5.511	-4.633
	(-0.59)	(-0.67)	(-0.64)	(-0.55)
DPSL	2.024	0.975	0.157	2.332
	(0.23)	(0.11)	(0.02)	(0.28)
DPSS	-8.285	-7.949	-6.954	-6.072
	(-0.92)	(-0.91)	(-0.78)	(-0.70)
French		5.066***	5.676***	6.005***
		(3.66)	(3.88)	(3.69)
AST		-6.732***	-7.733***	-7.063***
		(-5.17)	(-5.26)	(-4.39)
Kandinsky		0.336	-0.103	0.143
		(0.27)	(-0.08)	(0.11)
Period		-1.429***	-1.425***	-1.425***
		(-23.74)	(-23.42)	(-23.44)
Age			1.778***	1.880***
			(2.87)	(2.99)
Female			2.318*	2.488**
			(1.92)	(1.98)
GameTheory			-4.658***	-5.107***
			(-3.25)	(-3.51)
Semester			-0.368	-0.309
			(-0.79)	(-0.65)
Experience			-4.400***	-5.121***
			(-2.78)	(-2.98)
Risk Aversion			-0.799**	-0.769**
			(-2.23)	(-2.13)
TVLang - Neither				-2.929
				(-1.38)
- French				-1.431
				(-0.93)
WTPCPGE - Don't know				-20.44**
				(-2.07)
- Low				-17.03*
				(-1.70)
- Medium				-22.15**
				(-2.23)
- High				-22.81**
				(-2.30)

Table 4.6.a: Regression Results I (Multi-Level Model)

Arts				-1.900
				(-1.20)
Constant	65.52***	84.80***	53.64***	72.90***
	(11.36)	(14.65)	(3.93)	(4.38)
σ_M	3.008***	2.981***	2.986***	2.959***
	(41.72)	(41.38)	(41.18)	(40.54)
σ	3.512***	3.458***	3.460***	3.459***
	(359.64)	(354.11)	(350.81)	(350.68)
LogLike	-26538.9	-26253.7	-25771.9	-25764.1
N	5350	5350	5250	5250

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4.6.b: Regression Results I (Multi-Level Model)

treatments, the randomisation was conditional on the restriction that we eventually end up with similar numbers across treatments. If, say, the SL treatment occurs more often in the first sessions by chance, the probability of being in that treatment is lowered such that, in expectation, we arrive at an equal split across the 10 treatments and control. Remember that,

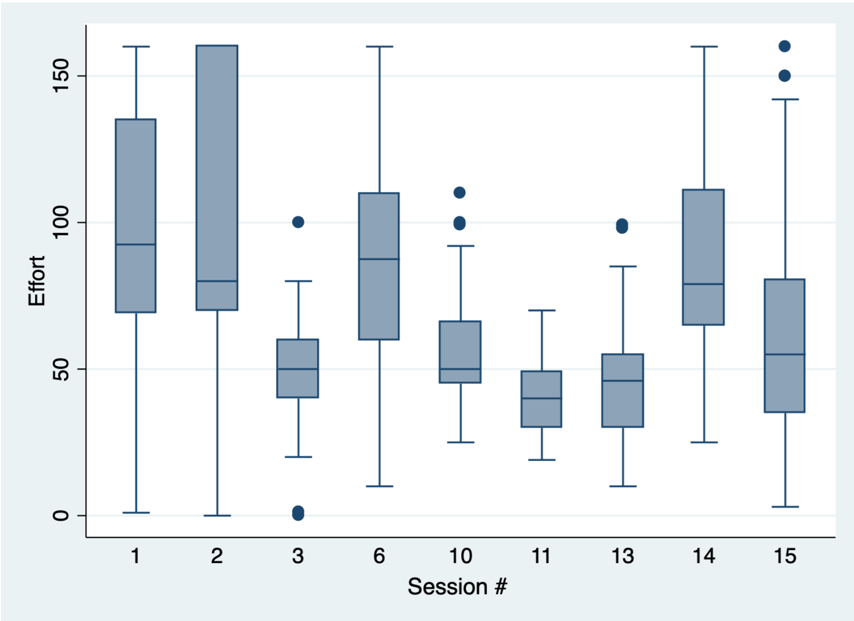


Figure 4.3.11 : Session Effects in Control Treatment

since treatments occur on the matching level, each session contains multiple treatments, i.e., while two matched subjects are in the control, two other matched subjects can be in any of the other treatments.

Figure 4.3.11 shows boxplots efforts in control treatments across sessions. Even accounting for the way we balanced the number of subjects in each treatment, as described above, the figure clearly shows, that sessions differ strongly in effort distributions. These session effects might be robust to the distribution of treatments and the inclusion of further controls. A simple multi-level model as presented earlier, might thus misrepresent the findings from this experiment.

To illustrate what these session effects do with the treatment dummies presented earlier, we run a multi-level model, allowing for the intercepts to vary, not only for each matching, but also for each session. As we see in Table 4.7 this has a strong effect on the treatment coefficients. None of them is significantly different from the baseline and even the between treatment comparisons are less significant. All the other results seem to be robust to these effects, and qualitative implications (signs and order of effects) are largely the same.[19]

Result 3.

Session effects are present and they strongly attenuate treatment effects.

These results need to be seen with caution as the number of sessions is relatively low (15). They are an indicator that treatment effects are not independent of factors that apparently differed across sessions, like the experimenter and unforeseen difficulties prior to some sessions.

Two particular reasons for the session effects come to mind. First, there is a long pause between the four blocks of sessions.[20] Within this period of time, there are changes of seasons and it is inevitable that there might be slight changes to the modus operandi as well. Second, the first six sessions are run by one of the authors, all the remaining sessions are run by employees of BSB who are also teaching staff, which might induce experimenter demand effect. Fréchette (2012) lists those reasons, among others, as culprits for why session effects can occur in the lab.

In the following model analysis we do not intend to incorporate these session effects. We take our results and the qualitative implications from the complete models in table 4.6 and 4.7 respectively, and suggest a model that offers an explanation for the observations and can produce hypotheses for follow up experiments.

4.4 A Spatial Model of Identity and Altruism

First, we discuss how making an additional identity dimension salient might affect an individual's perception of others in a stylised setting. Then we introduce our general model in which identity – a coordinate in social space – moderates the degree of altruism or spite. We apply it to the specific case of contest and relate it to the qualitative findings of our experiment and future experimental design.

[19]The SPDL treatment is negative throughout here, while in table 4.6 it is only negative in model (2).

[20]Between sessions six and seven there is a break of two months, between sessions eight and nine, there is a break of four months and between sessions 13 and 14 there is a break of 3 months.

Dep.Var.: Effort	(1)	(2)	(3)	(4)
SL	-5.941	-6.650	-3.775	-6.220
	(-0.69)	(-0.80)	(-0.45)	(-0.74)
DL	-5.819	-5.048	-5.520	-5.373
	(-0.70)	(-0.63)	(-0.69)	(-0.67)
SS	9.006	7.133	7.234	6.499
	(1.02)	(0.83)	(0.84)	(0.76)
DS	3.249	3.354	4.029	5.047
	(0.36)	(0.38)	(0.46)	(0.58)
SP	-11.39	-12.06	-11.04	-10.78
	(-1.25)	(-1.37)	(-1.24)	(-1.24)
SPDL	-1.991	-4.604	-2.853	-1.512
	(-0.23)	(-0.55)	(-0.34)	(-0.18)
SPDS	0.383	0.0662	-0.0289	1.897
	(0.04)	(0.01)	(-0.00)	(0.21)
DP	-5.916	-6.668	-6.443	-6.085
	(-0.69)	(-0.80)	(-0.77)	(-0.73)
DPSL	3.042	1.806	0.660	1.904
	(0.36)	(0.22)	(0.08)	(0.23)
DPSS	-11.33	-10.99	-10.26	-9.351
	(-1.31)	(-1.31)	(-1.21)	(-1.11)
French		4.914***	5.464***	5.886***
		(3.57)	(3.75)	(3.62)
AST		-6.875***	-7.862***	-7.183***
		(-5.30)	(-5.37)	(-4.47)
Kandinsky		0.291	-0.0671	0.110
		(0.24)	(-0.05)	(0.08)
Period		-1.429***	-1.425***	-1.425***
		(-23.74)	(-23.42)	(-23.44)
Age			1.727***	1.846***
			(2.80)	(2.94)
Female			2.234*	2.352*
			(1.86)	(1.88)
GameTheory			-4.755***	-5.184***
			(-3.33)	(-3.57)
Semester			-0.277	-0.266
			(-0.59)	(-0.55)
Experience			-4.497***	-5.157***
			(-2.84)	(-3.00)
Risk Aversion			-0.767**	-0.737**
			(-2.15)	(-2.04)
TVLang - Neither				-2.484
				(-1.17)
- French				-1.414
				(-0.92)
WTPCPGE - Don't know				-21.05*
				(-1.73)
- Low				-17.66
				(-1.44)
- Medium				-22.87*
				(-1.87)
- High				-23.41*
				(-1.92)

Table 4.7.a: Regression Results II (Multi-Level Model)

Arts				-1.912
				(-1.21)
Constant	67.20***	86.81***	56.46***	75.92***
	(11.06)	(14.28)	(4.11)	(4.23)
σ_M	2.215***	2.208***	2.231***	2.023***
	(6.28)	(6.50)	(6.68)	(5.05)
σ_S	2.916***	2.883***	2.885***	2.887***
	(36.25)	(35.90)	(35.77)	(35.69)
σ	3.512***	3.458***	3.460***	3.459***
	(359.64)	(354.11)	(350.81)	(350.68)
LogLike	-26536.7	-26251.2	-25769.2	-25762.5
N	5350	5350	5250	5250

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4.7.b: Regression Results II (Multi-Level Model)

4.4.1 A Formal Concept of (Multidimensional) Identity

Let there be individuals $i \in N_i = \{1, 2, ..., n\}$ who are described by a set of characteristics C that contains contributions to their identity. Since the order of elements in C is arbitrary, we say that the first c elements of C are salient in the given context.[21] For each dimension there exists a threshold value δ_k, where k is the index of the relevant characteristic.

Definition 1 (Social Group Radius)**.**
For any c, the social group radius for the active identity dimensions is

$$\delta^c = \sqrt{\sum_{k=1}^{c} \delta_{ik}^2} \tag{4.1}$$

Let $\boldsymbol{I}_i$ be player i's collection of these characteristics.

Definition 2 (Social Distance)**.**
The social distance is the function

$$d : \boldsymbol{I}_i \times \boldsymbol{I}_j \mapsto \mathbb{R}_+ \tag{4.2}$$

This distance measure captures social comparison.[22] In Shayo (2009) this distance measure is a weighted distance, where the so-called *salience weights* determine how much the respective dimension affects individual i's behaviour. Although we do not directly model weights that individuals attach to certain dimensions of identity we still model the fact that different

[21]Note that salience is not modelled in here. Just as in the experiment it is exogenously given which dimensions are active.

[22]For the rest of the argument we are working with the Euclidean metric. This is assuming real inputs. The argument goes through for metrics on other scales as well.

characteristics of one's own self and the opponent can influence behaviour to different degrees. This happens on the one hand through the discrete selection of each dimension into the active sets, which can be interpreted as putting weight $\omega_c = 0$ or $\omega_c = 1$ on some dimension $c \in \{1, ..., C\}$. On the other hand, the group thresholds δ_{ic} serve a similar purpose as only the joint information on distances and thresholds determines how similar individual i perceives j to be according to the salient dimensions of identity.
If an opponent is in the δ^c-ball centred at $\boldsymbol{I}_i$ with respect to the social distance, she is considered an in-group member. The formal definition of the in-group and the out-group is thus pinned down by the interplay of the distances between two identities $\boldsymbol{I}_i$ and $\boldsymbol{I}_j$ and the thresholds δ^c.

Definition 3 (In-Group and Out-Group)**.**
*From perspective of player i, we call the opponent j an **in-group member** if $d(\boldsymbol{I}_i, \boldsymbol{I}_j) \leq \delta^c$. The opponent is in the **out-group** if $d(\boldsymbol{I}_i, \boldsymbol{I}_j) > \delta^c$.*

Depending on whether individual i categorises j to be either in the in-group or the out-group, her behaviour in our model differs qualitatively compared to the standard equilibrium prediction.
Since our definition of in and out-group comparison is based on a spatial notion, we can easily imagine the case where we add an identity dimension to a one-dimensional identity.[23] Consider Figure 4.4.12 . Focus only on the abscissa first. Suppose some individual i's

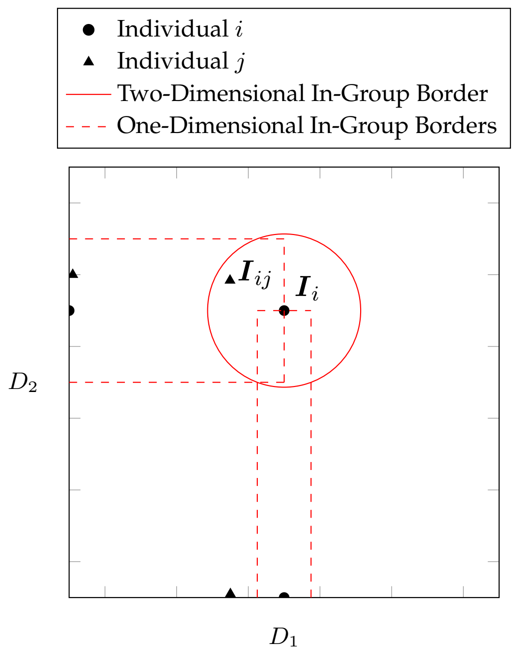


Figure 4.4.12 : Increasing Dimensionality of Active Identities

identity is determined solely by the corresponding dimension D_1. The dashed lines indicate

[23] Since distances as well as threshold values add up as a Euclidean sum this argument is without loss of generality. Formally, for each $\sqrt{\sum_{i=1}^{N} x_i^2} > \sqrt{\sum_{i=1}^{N} b_i^2}$ there exist $z, c \in \mathbb{R}$ such that $\sqrt{z^2 + \sum_{i=1}^{N} x_i^2} < \sqrt{c^2 + \sum_{i=1}^{N} b_i^2}$.

the social group radius for this single dimension, pinned down by δ_{iD_1}. As individual j's position is beyond this border, she is considered to be in the out-group of i with respect to D_1.

Now focus on the ordinate only. We see that i considers j to be an in-group member with respect to D_2 by the same argument.

Adding up both dimensions as specified above, each individual's position is determined by the two-dimensional coordinate on $D_1 \times D_2$. We see that individual j now belongs to i's in-group with respect to both identity dimensions, although she was in the out-group with respect only to D_1. We can easily construct the converse case. If we start out with D_2 and consider adding D_1, we will see that individual j moves much closer to the out-group of individual i.

This essentially illustrates what Gaertner et al. (1993) call a re-categorisation from the out-group (in-group) to the in-group (out-group).

Besides the concept of re-categorisation, figure 4.4.12 also gives us a means of decomposing the eventual categorisation into in-group and out-group. Think of someone who violates a specific in-group criterion. She can still be considered as in-group member, as all her other characteristics are perfectly in line with those of the group.[24] Conversely, if on many dimensions a person is seen as far away, one could still acknowledge her to have virtues without seeing her as an in-group member.

4.4.2 Social Distance and Social Preferences

Let there be two individuals. Each individual $i \in \mathcal{I} = \{1, 2\}$ has an identity that is described by a point $\boldsymbol{I}_i$ in the identity space as described above.[25] With the above metric and a shorthand on notation we have $d_{ij} = d(\boldsymbol{I}_i, \boldsymbol{I}_j)$. Note that the distance from i to j is assumed to be the same as the distance from j to i.

Denoting the action of player i by a_i and the corresponding material payoff by π_i, social preferences are then induced by that distance in a way that is consistent with Figure 4.4.12 .

Definition 4.1 (Identity-Induced Social Preferences)**.**
Social distance translates into other-regarding utility via

$$U_i\left(\boldsymbol{a}, \boldsymbol{I}_i, \boldsymbol{I}_j\right) = \pi_i\left(a_i, a_{-i}\right) + \sum_{j \neq i} \alpha_{ij} \pi_j(a_j, a_{-j})$$

[24] The argument in the graph would still go through if the border just widens close to the own coordinate on the y-axis, thus creating a vertical ellipse. This would make the formal statement more tedious.

[25] We change $\boldsymbol{I}_{ij}$ to $\boldsymbol{I}_j$ here as there are only two players and the experimental design is such that any salient identity is known to both subjects.

where

$$\alpha_{ij} = \frac{A_i \delta^c - S_i d_{ij}}{\delta^c + d_{ij}}$$

with $A_i, S_i \in \mathbb{R}_+$ *for all* $i \in \mathcal{I}$.

Note that, assuming $d_{ij} > 0$, we have $\alpha_{ij} \in (-S_i, A_i)$ for all $j \in \mathcal{I}$. The maximum level of altruism is reached when the distance is close to zero, corresponding to α_{ij} close to A_i, while the maximum level of spite is reached for $d_{ij} \to \infty$, for which $\alpha_{ij} \to -S$. The level where $\alpha_{ij} = 0$ corresponds to $\frac{d_{ij}}{\delta^c} = \frac{A_i}{S_i}$, which measures the threshold where individuals are neither acting altruistic or spiteful. Note that a very altruistic individual (A_i high and S_i low) would even act in an altruistic way towards out-group individuals as long as they are not 'too far away'. The opposite applies for a spiteful individual (A_i low and S_i high) who would even attack in-group members if they are closer to the out-group.

4.4.3 Application to Contest

We focus on a case with two players $i \in \{1, 2\}$. For the purpose of our experiment we assume that players always care more about their own payoff than about the other player's payoff. Thus, to ensure $\alpha_{ij} \in (-1, 1)$, we assume $A_i = S_i = 1$ for all i. This implies $\alpha_{ij} = \alpha_{ji} = \alpha$. This might well not be the case for real-world examples where the stakes are high and ethnic or religious identities have contributed to fierce fights over long periods of time. Also symmetry with respect to the α's might not be justified in some cases, as conflict between minorities and insurgents might well induce different levels of antagonism and in-group love.

Assuming linear costs, the payoff in a contest model is given by

$$\pi_i(x_i, x_j) = p(x_i, x_j)\, v_i - x_i$$

where $x_i \geq 0$ denotes player i's effort level, $v_i > 0$ is player i's valuation of the prize each player can obtain. The contest success function (CSF) is given by

$$p(x_i, x_j) = \begin{cases} \frac{x_i}{x_i + x_j} & \text{if } x_i + x_j > 0 \\ \frac{1}{2} & \text{else} \end{cases}$$

Within the framework developed before, utility obtains as

$$\begin{aligned} U_i(x_i, x_j, I_i, I_j) &= \pi_i(x_i, x_j) + \alpha(I_i, I_j)\, \pi_j(x_j, x_i) \\ &= \frac{x_i v_i + \alpha x_j v_j}{x_i + x_j} - x_i + \alpha x_j \end{aligned}$$

The best response functions obtain as

$$x_i^{\mathrm{BR}}(x_j) = \sqrt{x_j(v_i - \alpha v_j)} - x_j$$

The equilibrium is then given by

$$x_i = \frac{\tilde{v}_{ij}\tilde{v}_{\mathrm{ji}}}{\left(\tilde{v}_{ij} + \tilde{v}_{\mathrm{ji}}\right)^2}\tilde{v}_{ij}$$

where $\tilde{v}_{ij} = v_i - \alpha v_j$.

Our first assumption concerning differences and commonalities in an identity can be summarised as follows.

Assumption 1 (Differences and Commonalities in Identity)**.**
A salient identity induces $d_{ij} > \delta^c$ in case of a difference and $d_{ij} < \delta^c$ in case of a commonality.

Defining $B = \frac{\tilde{v}_{ij}\tilde{v}_{\mathrm{ji}}}{\left(\tilde{v}_{ij}+\tilde{v}_{\mathrm{ji}}\right)^2}$, we see that $x_i = B\tilde{v}_{ij}$ and $x_j = B\tilde{v}_{\mathrm{ji}}$.

For the special case of $v_i = v_j = v$, that is when the valuation of the prize is homogenous, we have

$$x^* = \frac{1}{4}(1+\alpha)v$$

We see that, depending on the players' identities, equilibrium can range from peaceful division of the prize ($\alpha = -1$) to full dissipation ($\alpha = 1$).
Figure 4.4.13 illustrates this result graphically. If social distance is great relative to the in-group border, i.e., if they perceive each other to be in their respective out-groups, the players end up in an equilibrium with higher efforts than in the purely selfish case ($\alpha = 0$). If they perceive each other to be sharing the same in-group, they reduce effort levels and dissipate less of the monetary value of the prize.

This model so far can explain how greater social distance can aggravate conflict, and lower social distance can reduce conflict. Our findings regarding higher efforts for the CPGE subjects and even higher efforts when these are matched with opponents of the same identity runs counter to the theoretical findings.
In the empirical section we argued that (a) more competitive students select into the CPGE and (b) that competition typically takes place within a vertical strata. We can model this through a greater losing loss (L^h) in the utility function when being matched with opponents of the same vertical identity. They might still have a loss when they are matched with opponents of a different vertical identity (L^l), but that loss is lower in magnitude ($L^h < L^l < 0$). This captures the idea that competition typically takes place within a certain level of vertical identities. Jobs are contested by workers of similar skill level, sports competitions are

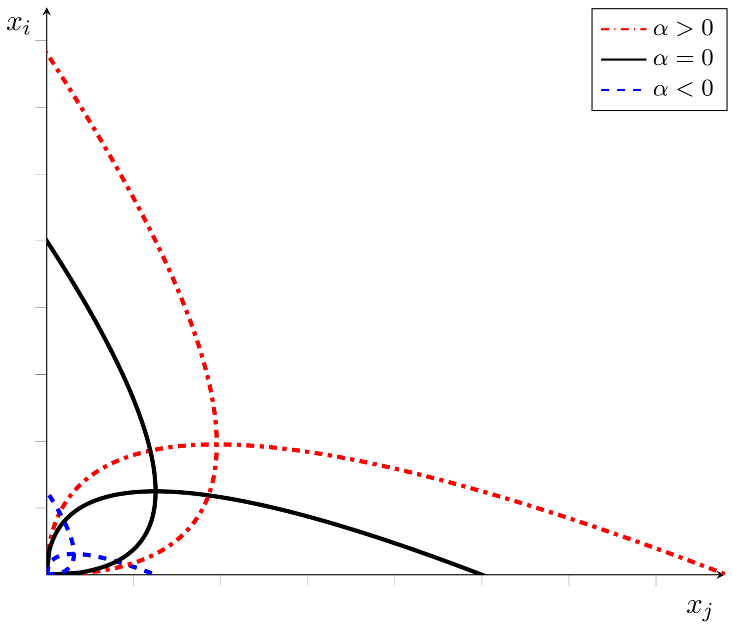


Figure 4.4.13 : Best Response Functions for Different Levels of α and homogenous L

organised in different divisions that show roughly the same performance and in the particular example of our experiment, students with similar grades and certificates tend to be more likely to compete for the same university places, internships and later jobs. Furthermore, we assume that the higher the level of the vertical identity in terms of its underlying hierarchy, the higher that loss ($L_{CPGE} > L_{AST}$).[26]

We add these to the utility function as the loss incurred when the opponent wins.

$$\pi_i'(x_i, x_j) = \pi_i(x_i, x_j) + p\left(x_j, x_i\right) L_i \tag{4.3}$$
$$= \frac{x_i v_i + \alpha x_j v_j - L_i}{x_i + x_j} - x_i + \alpha x_j \tag{4.4}$$

The best response functions change to

$$x_i^{\text{BR}}\left(x_j\right) = \sqrt{x_j(v_i - \alpha v_j - L_i)} - x_j. \tag{4.5}$$

This results in an equilibrium that is structurally identical to (4.3) for $\tilde{v}_{ij}' = \tilde{v}_{ij} - L_i = v_i - \alpha v_j - L_i$.

$$x_i = \frac{\tilde{v}_{ij}' \tilde{v}_{ji}'}{\left(\tilde{v}_{ij}' + \tilde{v}_{ji}'\right)^2} \tilde{v}_{ij}'$$

The other extremal case we can then discuss obtains for $\alpha = 0$ and some $L_i, L_j < 0$. If the two players have the same rank in the vertical identity, their loss function is increasing equilibrium efforts in a symmetric way. If player 1 has a higher rank than her opponent, we

[26]These assumptions are more mechanical than the assumptions about purely horizontal identity, but they correspond most naturally to our findings in the data.

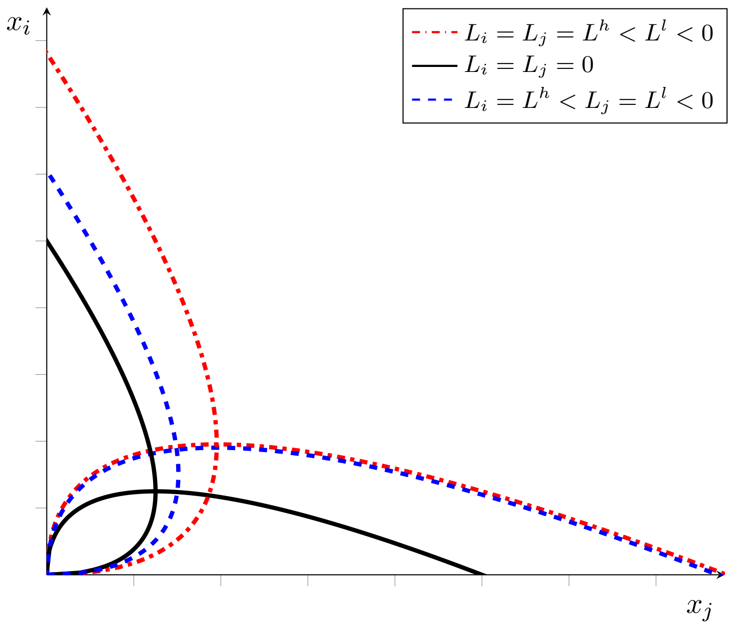


Figure 4.4.14 : Best Response Functions for Different Levels of L and homogenous α
Note: The blue dashed and the red dot-dashed line are supposed to lie on top of each other and are shifted for better legibility.

see that

$$x_1^* = S(v + \alpha - L_1) > S(v + \alpha - L_2) = x_2^*$$

and since both loss values are strictly negative and thus increasing $\tilde{v}_{ij}$ and $\tilde{v}_{ji}$,

$$x_1^* + x_2^* > 2x_{L=0}^*.$$

In terms of the treatments in our design, we consider six potential realisations for the parameters, which are given by $L_i \in \{L^h, L^l, 0\}$ for all $i \in \{1, 2\}$ with $L^h < L^l < 0$ and $\alpha \in \{\alpha_l, 0, \alpha_h\}$ with $-1 < \alpha_l < 0 < \alpha_h < 1$.

Identity Dimension	None	Language Same	Language Different	School Same	School Different
None	$\alpha = 0, L = 0$	$\alpha^l, L = 0$	$\alpha^h, L = 0$	α^l, L^h	α^h, L^h, L^l
Same Painting	$\alpha^l, L = 0$	–	$\alpha \in (0, \alpha^h)$, $L = 0$	–	$\alpha \in (0, \alpha^h)$, L^h, L^l
Different Painting	$\alpha^h, L = 0$	$\alpha \in (\alpha^l, 0)$, $L = 0$	–	$\alpha \in (\alpha^l, 0)$, L^h	–

Table 4.8: Model Parameters Corresponding to Experimental Design
Note: α is always assumed to be symmetric, L denotes symmetric parameters for L_i

Some of the comparisons depend on the magnitude of either α or L and how they compare. Other comparisons are relatively clear cut. According to the above model we have

- $x_{\alpha_l,L=0} > x_{\alpha_h,L=0}$
- $x_{\alpha=0,L^h} > x_{\alpha=0,L^l} > x_{\alpha=0,L=0}$
- $x_{\alpha=0,L^l}, x_{\alpha=0,L^h} \lesseqgtr x_{\alpha_h,L=0}$

Qualitatively, the second hypothesis is robust to allowing for a small, positive α. That would mean, that individuals from the same strata feel socially close, but that feeling is exceeded by the increase in competitiveness within strata.

4.5 The Model vs. the Data

The above model can easily reproduce behaviour observed in the pilot experiment whenever a single identity dimension is salient to the subjects. Differences in horizontal identities increase conflict relative to commonalities in these identities. Qualitatively, this is what we found for the minimal identity, which can also be categorised as horizontal.

Vertical identities trigger the loss function and thus increase efforts in general. If the two players are from different levels of the vertical hierarchy, the higher rank player is more aggressive than the lower rank player which is consistent with our finding that CPGE students bid more aggressively than AST students. Also, within the DS treatment, the CPGE students exerted more effort, but with the low number of independent observations, it needs to be shown in future studies whether this is robust.

When more than one dimension is activated, particularly, as in our design, with one commonality and one difference, comparison of equilibrium efforts is indeterminate in general. Only if α and L_i are sufficiently bound away from 0, this comparison gives clear answers. For the design of future experiments this means that identities should be strong enough, particularly when working with multiple identity dimensions. Pre-treatments in which subjects work together on a task as a way of strengthening the in-group identity might alleviate these problems as long as they are comparable across identity dimensions.

A few questions remain unanswered. The control group in the experiment exerts strikingly high efforts when compared to previous baselines and the other treatment groups. There are two candidate explanations we can come up with. As a first explanation, the session effects might be particularly strong in that group. This is supported by a correlation of

0.2246 between a dummy of being assigned to the control or not and the session number. Control treatments thus occurred more often in the second half of sessions where the clusters are stronger (consider Figure 4.3.11 again). As a second explanation, the information 'you are matched with another participant', which was displayed as a means of comparability between the experimental groups, might have induced overthinking in subjects. We do have data of a session where this message was not displayed for technical reasons, and differences between these two 'types' of control group are almost zero, but as there were other problems in this session, that comparison is not likely to be reliable.

4.6 Conclusion

We tested whether it is possible to classify identities into three different categories based on their effects in a conflict game. Also, we explored, whether commonalities in one identity dimension can mitigate conflict induced by another identity dimension where a difference is salient. The categories we defined are minimal identities and real identities that can be divided into horizontal and vertical identities, depending on whether the realisations of the identity can be ordered by an impartial third person. We have mild evidence that the effects of those different types of identity differ, but we lack statistical significance. Follow-up sessions of the experiment and further investigation of the data at hand can hopefully resolve these issues. To guide this future research we developed a model that suggests that differences in minimal and horizontal identities should increase conflict, while similarities should reduce it. For vertical identities we predicted that efforts generally increase. Also for vertical identities, commonalities increase efforts more strongly than commonalities since competition typically occurs within a social stratum.

Beside the follow ups for the experimental study, future directions point at a more complete theoretical investigation of vertical and horizontal identities. First attempts of applying similarity theory (Tversky, 1977) to the social space we defined here have been made. To structure our findings from the preliminary experiment carried out here and to generate hypotheses for future experiments, we introduced vertical identities in a fairly mechanical way. New models of identity may want to find a more endogenous way to create the distinction to horizontal identity.

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4.A Instructions

Since the instructions were displayed to the subjects in the respective language that they are taught in, we are first showing instructions in English and then in French. We keep translations consistent throughout the instructions and they do not differ dramatically in length, both in terms of words written and time needed to read through. Note though, that the experimenter did not read these instruction out loud, as subjects with different languages could have been present in each session.
They were first translated by one of the authors together with a native speaker from BSB and then checked by a French researcher at UEA.

4.A.1 English

GENERAL INSTRUCTIONS

This is an experiment in decision-making. If you follow the instructions carefully, you can earn more money depending both on your own decisions and on the decisions of others. These instructions and your decisions in this experiment are solely your private information. During the experiment you are not allowed to communicate with any of the other participants or with anyone outside the laboratory. Please switch off your mobile phone now. If you have any questions at any time during the course of this experiment, please raise your hand. An experimenter will assist you privately.

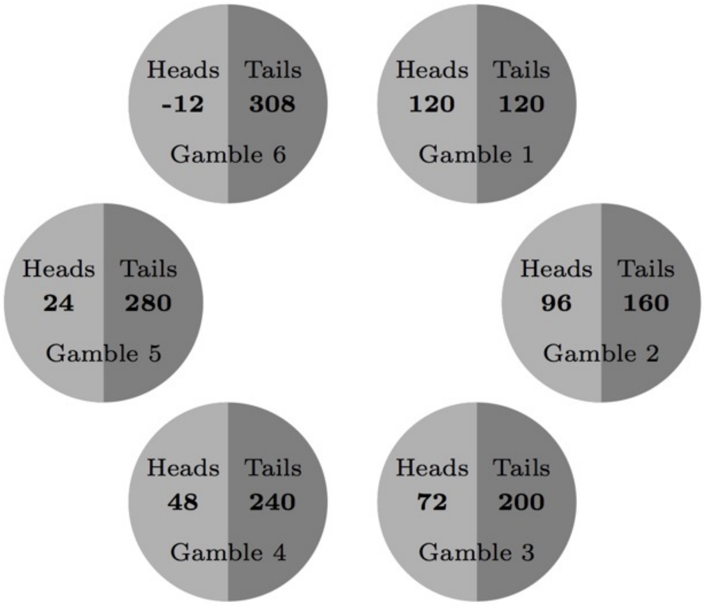
This experiment has **3 parts**. Your total earnings will be the sum of your earnings in all three parts. Your earnings in each part will be measured in tokens. At the end of the experiment you will be **paid in Euro** based on the exchange rate. In the end of the experiment your payments from all parts of the experiment are added up and you will be paid accordingly.

The exchange rate between tokens and Euro is the following:

1 token = 0.01 € = 1 Cent

INSTRUCTIONS FOR PART I

In this task, you will be asked to **chose from six different gambles** (as shown below). Each circle represents a different gamble from which you must **choose the one that you prefer.** Each circle is divided in half, with the two possible alternative payoffs from the gamble in the two halves of the respective circle.



Your payment for this task will be determined at the end of today's experiment. The computer will perform a random coin toss. If the outcome is a head, you will receive an amount of cents equal to the number in the light grey area of the circle. Alternatively, if the outcome is a tail, you will receive the amount of cents equal to the number shown in the dark grey area of the circle. Note that no matter which gamble you pick, each outcome has a 50% chance of occurring.

PART I QUIZ

Please answer the following questions to test your understanding of the task. You will proceed when you have answered both questions correctly. Please ask for help if you are unable to answer the questions correctly.

Q1) Suppose you chose Gamble 3 and the outcome of the coin toss is heads. How many cents would you receive from this task?

Q2) Suppose you chose Gamble 5 and the outcome of the coin toss is tails. How many cents would you receive from this task?

Please select the gamble of your choice by entering the corresponding number.

Once you click the 'Confirm' button, you cannot revise your choice anymore. So, please be sure of your choice before clicking on the 'Confirm' button.

INSTRUCTIONS FOR PART II

In **Part II** everyone will be shown **5 pairs of paintings** by two artists (Paul Klee and Wassily Kandinsky). You will be asked to choose which painting in each pair you prefer. You will then be classified into one of two groups, based on which artist you prefer.

Now please choose which painting you prefer by ticking the box under the corresponding painting from each pair. After everyone submits answers, you will be privately informed of which group you are in.

INSTRUCTIONS FOR PART III

Part III consists of **25 decision-making periods**. At the beginning of this stage, you will be randomly and anonymously matched with another participant. This matching will remain the same over all periods. Each period, you will be given an endowment of 160 tokens. You may bid for a reward of 160 additional tokens. You may bid any whole number between 0 and 160. Irrespective of whether you get the reward or not, your bid is gone.

Your bid and the bid of the participant you are matched with will determine the probability of winning the reward for you and your opponent. This probability will be calculated as

Probability=Your bid/(Your bid + Other's bid)

A random draw according to this probability will then determine who won the reward. Your earnings in each round you win will be

Reward+Endowment-Bid

Your earnings in each round you lose will be

Endowment-Bid

Your endowment does not carry over to future periods.

In the end, **5 periods** will be randomly chosen to determine your total earnings and thus your payment.

An Example (for illustrative purposes only)

Assume participant 1 bids 30 tokens and participant 2 bids 45 tokens.

Therefore, the computer assigns 30 lottery tickets to participant 1 and 45 lottery tickets to participant 2. Then the computer randomly draws **one lottery ticket out of 75 (30 + 45)**.

As you can see, participant 2 has the **highest chance** of receiving the reward: **0.60 = 45/75** and participant 1 has a **0.40 = 30/75 chance** of receiving the reward.

Assume that the computer assigns the reward to participant 1, then the **earnings of participant 1 for the period are 290 = 160 + 160 – 30**, since the reward is 160 tokens and the cost of the bid is 30. Similarly, the **earnings of participant 2 are 115 = 160 – 45**.

To demonstrate your understanding of the task in this stage, please answer the following questions. You will proceed when you have answered both questions correctly. Please ask for help if you are unable to answer the questions correctly.

4.A.2 French

CONSIGNES GENERALES

C'est une expérience basée sur la prise de décision. Si vous suivez attentivement les instructions, vous pouvez gagner plus d'argent en fonction de vos propres décisions et de celles des autres. Ces instructions et les décisions que vous prenez dans le cadre de cette expérience ne sont que des renseignements personnels vous concernant. Pendant l'expérience, vous n'êtes pas autorisé à communiquer avec les autres participants ou avec quiconque en dehors du laboratoire. Veuillez éteindre votre téléphone portable maintenant. Si vous avez des questions au cours de cette expérience, veuillez lever la main. Un expérimentateur vous assistera en privé.

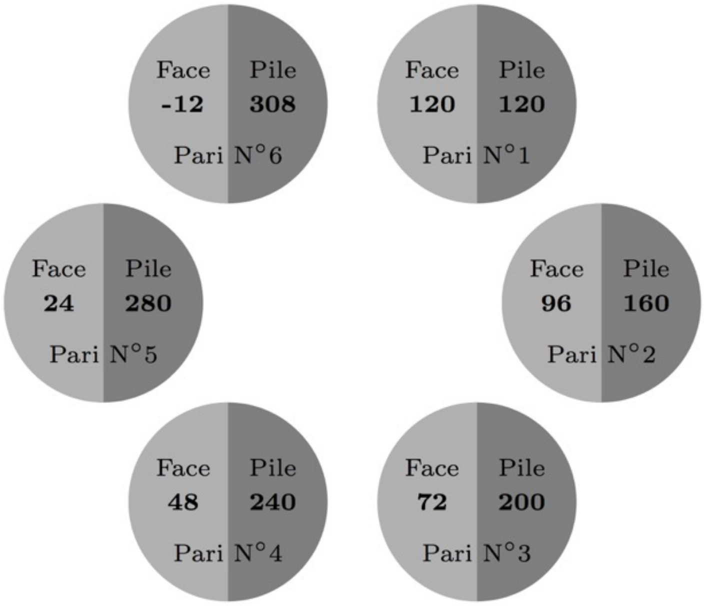
Cette expérience comporte **3 parties**. Le total de vos gains correspondra à la somme de vos gains dans les trois parties. Vos gains dans chaque partie seront mesurés en jetons. À la fin de l'expérience, vous serez **payé en EUROS** sur la base du taux de change. À la fin de l'expérience, vos paiements de toutes les parties de l'expérience sont additionnés et vous serez payés en conséquence.

Le taux de change entre les jetons et l'euro est le suivant:

1 jeton = 0.01€ = 1 Centime

CONSIGNES POUR LA PARTIE I

Dans cette tâche, vous devrez **choisir parmi six jeux différents** (comme illustré ci-dessous). Chaque cercle représente un pari diffèrent parmi lequel vous devez **choisir celui que vous préférez**. Chaque cercle est divisé en deux, avec les deux variantes possibles de gains du pari dans les deux moitiés du cercle respectif.



Votre paiement pour cette tâche sera déterminé à la fin de l'expérience d'aujourd'hui. L'ordinateur effectuera un tirage au sort des pièces de monnaie. Si le résultat est face, vous recevrez un montant de centimes égal au nombre dans la zone gris clair du cercle. Alternativement, si le résultat tombe sur pile, vous recevrez une quantité de centimes égale au nombre indiqué dans la zone gris foncé du cercle. Notez que peu importe le jeu que vous choisissez, chaque résultat a 50% de chance de se produire.

PARTIE I QUIZ

Veuillez répondre aux questions suivantes pour tester votre compréhension de la tâche à effectuer. Vous pourrez commencer une fois que vous aurez répondu correctement aux deux questions. Veuillez demander de l'aide si vous ne pouvez pas répondre correctement aux questions.

Une fois que vous avez cliqué sur "Confirmer", vous ne pouvez plus modifier votre choix. Veuillez donc vous assurer de votre choix avant de cliquer sur "Confirmer".

CONSIGNES POUR LA PARTIE II

Dans la **partie II**, chacun pourra voir **5 paires de tableaux** de deux artistes (Paul Klee et Wassily Kandinsky) Il vous sera demandé de choisir quelle peinture de chaque paire vous préférez. Vous serez ensuite classés dans l'un des deux groupes, en fonction de l'artiste que vous préférez.

Veuillez maintenant choisir quel tableau vous préférez en cochant la case sous le tableau correspondant de chaque paire. Une fois que tout le monde aura répondu, vous serez informés en privé du groupe auquel vous appartenez.

CONSIGNES POUR LA PARTIE III

La Partie III comprend **25 périodes de prise de décision.** Au début de cette étape, vous serez jumelés de façon aléatoire et anonyme avec un autre participant. **Cette association restera la même pour toutes les périodes**. À chaque période, vous recevez une dotation de 160 jetons. Vous pouvez **enchérir pour obtenir une récompense de 160 jetons supplémentaires**. Vous pouvez enchérir sur n'importe quel nombre entier entre 0 et 160. Peu importe **que vous obteniez la récompense ou non, votre enchère a disparu**.

Votre enchère et celle du participant avec lequel vous êtes associé déterminera la probabilité de gagner la récompense pour vous et le participant avec lequel vous êtes associé. Cette probabilité sera calculée comme suit :

Probabilité=Votre enchère/ (Votre enchère + L'enchère d'autre)

Un tirage au sort selon cette probabilité déterminera alors qui a gagné la récompense. Vos gains à chaque tour que vous gagnerez seront les suivants :

Prime + dotation - enchère

Vos gains à chaque tour que vous perdez seront les suivants :

Dotation – Enchère

Votre dotation ne se reporte pas aux périodes futures.

En fin de compte, **5 périodes** seront choisies au hasard pour déterminer le total de vos gains et donc votre paiement.

Un exemple (À titre indicatif seulement)

Supposons que le participant 1 offre 30 jetons et le participant 2 offres 45 jetons.

Par conséquent, l'ordinateur attribue 30 billets de loterie au participant 1 et 45 billets de loterie au participant 2. L'ordinateur tire au hasard **un billet de loterie sur 75 (30 + 45).**

Comme vous pouvez le voir, le participant 2 a **la plus grande chance** de recevoir la récompense : **45 sur 75** et le participant 1 a **30 sur 75** de chance de recevoir la récompense.

Supposons que l'ordinateur attribue la récompense au participant 1, alors **les gains du participant 1 pour la période seront 290 = 160 + 160 – 30**, puisque la récompense est de 160 jetons et que le coût de l'enchère est de 30. De même, **les gains du participant 2 sont de 115 = 160 – 45.**

Afin de démontrer que vous comprenez le travail à effectuer, veuillez répondre aux questions suivantes. Vous procèderez une fois que vous aurez répondu correctement aux deux questions. Veuillez demander de l'aide si vous ne pouvez pas répondre correctement aux questions.

4.B Regression Tables

The first table checks for the individual inclusion of the attitude variables to see which has the largest impact on the remaining coefficients. We see that the largest change occurs when including WTPCPGE. Compared to model (1), this model sees the largest changes in magnitude in the treatment variables. The coefficients on French, AST, Female, GameTheory and Experience attenuate, but they remain significant. The coefficients on Age and Risk Aversion increase slightly, also remaining statistically significant.
Since the baseline category of WTPCPGE is 'no response', the interpretation is that subjects in session 1 where technical difficulties occurred exerted more effort than those that actually answered the questionnaire.

Table 4.10 investigates whether subjects with a CPGE degree did better in terms of period payoffs than AST subjects. While in model (2) this seems to be very slightly the case, the magnitude and statistical significance attenuate as we include the standard controls.
Table 4.11 finally check whether the winning probabilities are higher for CPGE students. In case they do not care as much about payoff as they do about winning, this might still signal a higher ability and then provide an alternative explanation to our interpretation, based on selection.
The AST coefficient is close to zero and insignificant, so we can reject the hypothesis that CPGE students are systematically better at winning than AST students.

Dep.Var.: Effort	(1)	(2)	(3)	(4)
SL	-3.775	-3.393	-6.954	-3.340
	(-0.45)	(-0.40)	(-0.83)	(-0.40)
DL	-5.520	-5.309	-5.537	-5.493
	(-0.69)	(-0.66)	(-0.69)	(-0.68)
SS	7.234	7.369	6.847	6.601
	(0.84)	(0.85)	(0.80)	(0.77)
DS	4.029	3.727	5.459	3.934
	(0.46)	(0.42)	(0.62)	(0.45)
SP	-11.04	-11.29	-10.62	-11.13
	(-1.24)	(-1.27)	(-1.21)	(-1.25)
SPDL	-2.853	-2.780	-1.139	-3.339
	(-0.34)	(-0.33)	(-0.14)	(-0.40)
SPDS	-0.0289	0.00119	2.598	-1.246
	(-0.00)	(0.00)	(0.29)	(-0.14)
DP	-6.443	-6.832	-5.872	-6.523
	(-0.77)	(-0.81)	(-0.71)	(-0.78)
DPSL	0.660	0.851	1.567	0.809
	(0.08)	(0.10)	(0.19)	(0.10)
DPSS	-10.26	-10.61	-8.835	-10.51
	(-1.21)	(-1.25)	(-1.05)	(-1.24)
French	5.464***	6.326***	4.919***	5.663***
	(3.75)	(3.95)	(3.35)	(3.88)
AST	-7.862***	-7.975***	-6.975***	-8.015***
	(-5.37)	(-5.41)	(-4.37)	(-5.46)
Kandinsky	-0.0671	0.0961	0.0584	-0.178
	(-0.05)	(0.07)	(0.05)	(-0.14)
Period	-1.425***	-1.425***	-1.425***	-1.425***
	(-23.42)	(-23.42)	(-23.43)	(-23.42)
Age	1.727***	1.784***	1.842***	1.639***
	(2.80)	(2.88)	(2.96)	(2.64)
Female	2.234*	2.550**	2.326*	1.992
	(1.86)	(2.07)	(1.93)	(1.64)
GameTheory	-4.755***	-4.939***	-4.775***	-5.161***
	(-3.33)	(-3.44)	(-3.34)	(-3.57)
Semester	-0.277	-0.344	-0.161	-0.296
	(-0.59)	(-0.72)	(-0.34)	(-0.63)
Experience	-4.497***	-5.009***	-4.134**	-5.059***
	(-2.84)	(-3.07)	(-2.57)	(-3.13)
Risk Aversion	-0.767**	-0.745**	-0.771**	-0.775**
	(-2.15)	(-2.08)	(-2.15)	(-2.17)
Neither		-3.013		
		(-1.45)		
French		-1.478		
		(-0.98)		
Don't know			-22.08*	
			(-1.78)	
Low			-18.07	
			(-1.45)	
Medium			-23.20*	
			(-1.87)	
High			-24.46**	
			(-1.97)	

Table 4.9.a: Regression Results III: Individual Inclusion of Attitude Controls

Arts				-2.592*
				(-1.72)
Constant	56.46***	55.99***	74.38***	60.60***
	(4.11)	(4.08)	(4.14)	(4.35)
σ_M	2.231***	2.196***	2.067***	2.210***
	(6.68)	(6.30)	(5.48)	(6.49)
σ_S	2.885***	2.891***	2.883***	2.887***
	(35.77)	(35.76)	(35.74)	(35.78)
σ	3.460***	3.459***	3.459***	3.459***
	(350.81)	(350.78)	(350.72)	(350.78)
LogLike	-25769.2	-25768.1	-25764.3	-25767.7
N	5350	5350	5250	5250

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4.9.b: Regression Results III: Individual Inclusion of Attitude Controls

Dep.Var.: Period Earnings	(1)	(2)	(3)	(4)
SL	5.941	5.988	4.539	5.171
	(0.69)	(0.71)	(0.53)	(0.60)
DL	5.819	5.866	4.429	4.607
	(0.70)	(0.72)	(0.54)	(0.57)
SS	-9.006	-7.858	-9.072	-8.747
	(-1.02)	(-0.90)	(-1.03)	(-1.00)
DS	-3.249	-3.880	-4.608	-5.993
	(-0.36)	(-0.44)	(-0.51)	(-0.68)
SP	11.39	13.37	11.73	10.57
	(1.25)	(1.49)	(1.29)	(1.19)
SPDL	1.991	3.878	-0.775	-1.033
	(0.23)	(0.46)	(-0.09)	(-0.12)
SPDS	-0.383	-0.378	-1.459	-4.634
	(-0.04)	(-0.04)	(-0.16)	(-0.51)
DP	5.916	7.003	5.633	3.013
	(0.69)	(0.83)	(0.66)	(0.36)
DPSL	-3.042	-2.135	-3.308	-5.078
	(-0.36)	(-0.26)	(-0.40)	(-0.62)
DPSS	11.33	11.23	9.008	8.199
	(1.31)	(1.32)	(1.04)	(0.96)
French		-6.376**	-5.392*	-4.689
		(-2.21)	(-1.82)	(-1.44)
AST		4.305	3.462	2.838
		(1.51)	(1.11)	(0.84)
Kandinsky		2.461	2.043	2.217
		(0.90)	(0.73)	(0.79)
Period		1.429***	1.437***	1.437***
		(9.37)	(9.34)	(9.34)
Age			0.562	0.249
			(0.42)	(0.18)
Female			1.873	2.327
			(0.68)	(0.82)
GameTheory			1.355	0.987
			(0.42)	(0.30)
Semester			-0.654	-0.569
			(-0.66)	(-0.57)
Experience			-1.541	0.00425
			(-0.42)	(0.00)
Risk Aversion			0.337	0.210
			(0.43)	(0.27)
TVLang - Neither				-0.462
				(-0.10)
- French				-1.868
				(-0.56)
WTPCPGE - Don't know				19.38
				(1.52)
- Low				24.89*
				(1.87)
- Medium				27.01**
				(2.10)
- High				20.64
				(1.61)

Table 4.10.a: Regression Results IV: Final Earnings

Arts				2.161
				(0.63)
Constant	172.8***	153.8***	143.9***	129.3***
	(28.44)	(22.57)	(5.18)	(4.21)
σ_M	2.215***	2.174***	2.227***	2.070***
	(6.28)	(6.07)	(6.30)	(5.16)
σ_S	2.722***	2.698***	2.687***	2.661***
	(22.98)	(21.94)	(21.12)	(20.42)
σ	4.396***	4.388***	4.387***	4.387***
	(450.17)	(449.12)	(444.59)	(444.49)
LogLike	-31172.0	-31125.1	-30542.1	-30538.2
N	5350	5350	5250	5250

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4.10.b: Regression Results IV: Final Earnings

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